

15 Afgeleiden en primitieven

Voorkennis Lengten van lijnstukken

Bladzijde 91

$$\begin{aligned}
 \text{1 } f(x) = 7\frac{1}{2} \text{ geeft } 8\sqrt{x} - 2x &= 7\frac{1}{2} \\
 8\sqrt{x} &= 2x + 7\frac{1}{2} \\
 16\sqrt{x} &= 4x + 15 \\
 256x &= 16x^2 + 120x + 225 \\
 16x^2 - 136x + 225 &= 0 \\
 D &= (-136)^2 - 4 \cdot 16 \cdot 225 = 4096 \\
 x &= \frac{136 \pm 64}{32} = 6\frac{1}{4} \vee x = \frac{136 - 64}{32} = 2\frac{1}{4}
 \end{aligned}$$

$$\text{Dus } EF = 6\frac{1}{4} - 2\frac{1}{4} = 4.$$

Bladzijde 92

- 2 Rechts van het snijpunt van de grafieken ligt de grafiek van g boven de grafiek van f , dus geldt dan $CD = g(p) - f(p)$.

$$\begin{aligned}
 CD = 8 \text{ geeft } p - 3 - (8\sqrt{p} - 2p) &= 8 \\
 p - 3 - 8\sqrt{p} + 2p &= 8 \\
 3p - 11 &= 8\sqrt{p} \\
 9p^2 - 66p + 121 &= 64p \\
 9p^2 - 130p + 121 &= 0 \\
 D &= (-130)^2 - 4 \cdot 9 \cdot 121 = 12544 \\
 p &= \frac{130 \pm 112}{18} = 13\frac{4}{9} \vee p = \frac{130 - 112}{18} = 1
 \end{aligned}$$

$$p = 1 \text{ voldoet niet, dus } p = 13\frac{4}{9}.$$

$$\begin{aligned}
 \text{3 a } A(0, -4) \text{ en } k: y &= -4 \\
 f(x) = -4 \text{ geeft } \frac{1}{2}x^2 - 2x - 4 &= -4 \\
 \frac{1}{2}x^2 - 2x &= 0 \\
 \frac{1}{2}x(x - 4) &= 0 \\
 x = 0 \vee x &= 4
 \end{aligned}$$

Dus $B(4, -4)$.

$$\begin{aligned}
 g(x) = -4 \text{ geeft } -\frac{1}{4}x^2 + 3x - 4 &= -4 \\
 -\frac{1}{4}x^2 + 3x &= 0 \\
 \frac{1}{4}x(-x + 12) &= 0 \\
 x = 0 \vee x &= 12
 \end{aligned}$$

Dus $C(12, -4)$.

$$A(0, -4) \text{ en } B(4, -4) \text{ geeft } AB = 4 - 0 = 4$$

$$B(4, -4) \text{ en } C(12, -4) \text{ geeft } BC = 12 - 4 = 8$$

$$\text{Dus } AB : BC = 4 : 8 = 1 : 2.$$

- b $DE = 8$, dus $|f(p) - g(p)| = 8$

$$\begin{aligned}
 \left| \frac{1}{2}p^2 - 2p - 4 - (-\frac{1}{4}p^2 + 3p - 4) \right| &= 8 \\
 \left| \frac{1}{2}p^2 - 2p - 4 + \frac{1}{4}p^2 - 3p + 4 \right| &= 8 \\
 \left| \frac{3}{4}p^2 - 5p \right| &= 8 \\
 \frac{3}{4}p^2 - 5p = 8 \vee \frac{3}{4}p^2 - 5p &= -8 \\
 \frac{3}{4}p^2 - 5p - 8 = 0 &\quad \vee \quad \frac{3}{4}p^2 - 5p + 8 = 0 \\
 D = (-5)^2 - 4 \cdot \frac{3}{4} \cdot -8 &= 49 \quad D = (-5)^2 - 4 \cdot \frac{3}{4} \cdot 8 = 1 \\
 p = \frac{5 \pm 7}{1\frac{1}{2}} = 8 \vee p = \frac{5 - 7}{1\frac{1}{2}} &= -1\frac{1}{3} \vee p = \frac{5 \pm 1}{1\frac{1}{2}} = 4 \vee p = \frac{5 - 1}{1\frac{1}{2}} = 2\frac{2}{3}
 \end{aligned}$$

$$\text{Dus } DE = 8 \text{ voor } p = -1\frac{1}{3} \vee p = 2\frac{2}{3} \vee p = 4 \vee p = 8.$$

4 a $f(x) = x e^x$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = e^x + x e^x$

b $f'(x) = 0$ geeft $e^x + x e^x = 0$

$$e^x(1+x) = 0$$

$$x = -1$$

$$f(-1) = -\frac{1}{e}, \text{ dus } A\left(-1, -\frac{1}{e}\right) \text{ en } k: y = -\frac{1}{e}.$$

$$g(x) = -\frac{1}{e} \text{ geeft } 2x - 1 = -\frac{1}{e}$$

$$2x = 1 - \frac{1}{e}$$

$$x = \frac{1}{2} - \frac{1}{2e}$$

$$\text{Dus } B\left(\frac{1}{2} - \frac{1}{2e}, -\frac{1}{e}\right).$$

$$A\left(-1, -\frac{1}{e}\right) \text{ en } B\left(\frac{1}{2} - \frac{1}{2e}, -\frac{1}{e}\right) \text{ geeft } AB = \frac{1}{2} - \frac{1}{2e} - (-1) = 1\frac{1}{2} - \frac{1}{2e}.$$

c $f(p) - g(p) = 1$ geeft $p e^p - (2p - 1) = 1$

$$p e^p - 2p + 1 = 1$$

$$p e^p - 2p = 0$$

$$p(e^p - 2) = 0$$

$$p = 0 \vee e^p = 2$$

$$p = 0 \vee p = \ln(2)$$

15.1 Lijnstukproblemen

Bladzijde 93

1 a $AB : BC = 1 : 2$, dus $BC = 2AB$.

$$AB = x_B = p$$

$$x_C = AC = AB + BC = AB + 2AB = 3AB = 3p$$

b $y_B = f(p) = e^p$

$$y_C = g(p) = e^{3p-3}$$

$$y_B = y_C, \text{ dus } e^p = e^{3p-3}.$$

$$\text{Dit geeft } p = 3p - 3$$

$$-2p = -3$$

$$p = 1\frac{1}{2}$$

c $q = f(p) = f(1\frac{1}{2}) = e^{1\frac{1}{2}} = e\sqrt{e}$

2 a Vermenigvuldigen van beide leden van $e^p = 3 e^{-3p}$ met e^{3p} geeft $e^{2p} = 3$.

b $e^{-\frac{1}{2} \cdot \ln(3)} = (e^{\ln(3)})^{-\frac{1}{2}} = 3^{-\frac{1}{2}}$

Bladzijde 94

3 Stel $x_B = p$, dan is $x_C = 2p$.

$$f(p) = f(2p) \text{ geeft } 6p e^p = 12p e^{2p}$$

$$6p = 0 \vee e^p = 2 e^{2p}$$

$$p = 0 \vee e^p = 2$$

$$\text{vold. niet } p = \ln(2)$$

$$q = f(p) = f(\ln(2)) = 6 \ln(2) \cdot e^{-\ln(2)} = 6 \ln(2) \cdot 2^{-1} = 3 \ln(2)$$

4 a Stel $x_B = p$, dan is $x_C = 3p$.

$$f(p) = g(3p) \text{ geeft } \ln(p) = \ln(3p - 3)$$

$$p = 3p - 3$$

$$-2p = -3$$

$$p = 1\frac{1}{2}$$

$$q = f(p) = f(1\frac{1}{2}) = \ln(1\frac{1}{2})$$

$$\begin{aligned} \text{b } & \left. \begin{aligned} y_F &= f(r) \\ y_E &= g(r) \\ y_F &= 2 \cdot y_E \end{aligned} \right\} f(r) = 2 \cdot g(r) \\ f(r) &= 2 \cdot g(r) \text{ geeft } \ln(r) = 2 \cdot \ln(r-3) \\ \ln(r) &= \ln((r-3)^2) \\ r &= (r-3)^2 \\ r &= r^2 - 6r + 9 \\ r^2 - 7r + 9 &= 0 \\ D &= (-7)^2 - 4 \cdot 1 \cdot 9 = 13 \\ r &= \frac{7 + \sqrt{13}}{2} \vee r = \frac{7 - \sqrt{13}}{2} \\ & \text{vold. niet} \end{aligned}$$

$$\text{Dus } r = 3\frac{1}{2} + \frac{1}{2}\sqrt{13}.$$

$$\begin{aligned} \text{5 a } & q = f(p) = g(p+3) \text{ geeft } \ln(p) = 2 \ln(p+3) - 3 \\ & \text{Voer in } y_1 = \ln(x) \text{ en } y_2 = 2 \ln(x+3) - 3. \\ & \text{De optie snijpunt geeft } x \approx 0,67 \text{ en } y \approx -0,40 \text{ en ook } x \approx 13,41 \text{ en } y \approx 2,60. \\ & \text{Dus } q \approx -0,40 \vee q \approx 2,60. \end{aligned}$$

$$\begin{aligned} \text{b } & f(x) = g(x) \text{ geeft } \ln(x) = 2 \ln(x) - 3 \\ & \ln(x) = 3 \\ & x = e^3 \end{aligned}$$

$$\text{Dus } S(e^3, 3).$$

$$\begin{aligned} \text{c } & CD = g(r) - f(r) = 2 \ln(r) - 3 - \ln(r) = \ln(r) - 3 \\ & CE = f(r) = \ln(r) \\ & \frac{CD}{CE} = \frac{\ln(r) - 3}{\ln(r)} \\ \lim_{r \rightarrow \infty} \frac{\ln(r) - 3}{\ln(r)} &= \lim_{\ln(r) \rightarrow \infty} \frac{\ln(r) - 3}{\ln(r)} = \lim_{\ln(r) \rightarrow \infty} \frac{1 - \frac{3}{\ln(r)}}{1} = \frac{1 - 0}{1} = 1 \\ & \text{Dus deze grenswaarde is 1.} \end{aligned}$$

Bladzijde 95

$$\begin{aligned} \text{6 a } & f(x) = \sin^2(x) - \frac{1}{2} \cos(x) \text{ geeft } f'(x) = 2 \sin(x) \cos(x) + \frac{1}{2} \sin(x) \\ & f'(x) = 0 \text{ geeft } 2 \sin(x) \cos(x) + \frac{1}{2} \sin(x) = 0 \end{aligned}$$

$$\sin(x)(2 \cos(x) + \frac{1}{2}) = 0$$

$$\sin(x) = 0 \vee \cos(x) = -\frac{1}{4}$$

$$x = k \cdot \pi \vee \cos(x) = -\frac{1}{4}$$

$$x = k \cdot \pi \text{ met } x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \pi \vee x = 2\pi$$

$$f(\pi) = \frac{1}{2}, \text{ dus } T(\pi, \frac{1}{2}) \text{ en } k: x = \pi.$$

$$\text{Voor symmetrie in } k \text{ moet gelden } f(\pi - p) = f(\pi + p).$$

$$f(\pi - p) = \sin^2(\pi - p) - \frac{1}{2} \cos(\pi - p) = (\sin(p))^2 + \frac{1}{2} \cos(p) = \sin^2(p) + \frac{1}{2} \cos(p)$$

$$f(\pi + p) = \sin^2(\pi + p) - \frac{1}{2} \cos(\pi + p) = (-\sin(p))^2 + \frac{1}{2} \cos(p) = \sin^2(p) + \frac{1}{2} \cos(p)$$

$$f(\pi - p) = f(\pi + p), \text{ dus de grafiek van } f \text{ is symmetrisch in } k.$$

$$\text{b } d(T, AB) = f(x_T + \frac{1}{2}) - y_T = f(\pi + \frac{1}{2}) - \frac{1}{2} = \sin^2(\pi + \frac{1}{2}) - \frac{1}{2} \cos(\pi + \frac{1}{2}) - \frac{1}{2} \approx 0,17$$

$$\begin{aligned} \text{7 a } & \text{Stel } x_B = p, \text{ dan is } x_C = p + 2. \\ & f(p) = g(p+2) \text{ geeft } 3^p = 10 - 3^p \\ & 2 \cdot 3^p = 10 \\ & 3^p = 5 \\ & q = f(p) = 3^p = 5 \end{aligned}$$

- b** Stel $x_B = p$, dan is $x_C = 2p$.

$$\begin{aligned} f(p) = g(2p) \text{ geeft } 3^p &= 10 - 3^{2p-2} \\ 3^{2p-2} + 3^p - 10 &= 0 \\ 3^{-2} \cdot 3^{2p} + 3^p - 10 &= 0 \\ \frac{1}{9} \cdot 3^{2p} + 3^p - 10 &= 0 \\ \frac{1}{9} \cdot (3^p)^2 + 3^p - 10 &= 0 \\ \text{Stel } 3^p &= u. \\ \frac{1}{9}u^2 + u - 10 &= 0 \\ u^2 + 9u - 90 &= 0 \\ (u - 6)(u + 15) &= 0 \\ u = 6 \vee u = -15 \\ 3^p = 6 \vee 3^p &= -15 \end{aligned}$$

$$q = f(p) = 3^p = 6$$

- c** $g(r) = 2 \cdot f(r)$ geeft $10 - 3^{r-2} = 2 \cdot 3^r$
- $$\begin{aligned} 10 - 3^{-2} \cdot 3^r &= 2 \cdot 3^r \\ 10 - \frac{1}{9} \cdot 3^r &= 2 \cdot 3^r \\ 2\frac{1}{9} \cdot 3^r &= 10 \\ 3^r &= 4\frac{14}{19} \\ r &= {}^3\log(4\frac{14}{19}) \end{aligned}$$

8 $rc_k = \frac{y_A}{x_A} = \frac{y_B}{x_B}$

Stel $x_A = p$. Dan is $y_A = p^2 - 5p + 6$.

Uit $OA : AB = 1 : 5$ volgt $x_B = 6x_A$.

Dit geeft $x_B = 6p$ en $y_B = 36p^2 - 30p + 6$.

$$\begin{aligned} \frac{y_A}{x_A} = \frac{y_B}{x_B} \text{ geeft } \frac{p^2 - 5p + 6}{p} &= \frac{36p^2 - 30p + 6}{6p} \\ \frac{6p^2 - 30p + 36}{6p} &= \frac{36p^2 - 30p + 6}{6p} \\ 6p^2 - 30p + 36 &= 36p^2 - 30p + 6 \\ 30p^2 &= 30 \\ p^2 &= 1 \\ p &= 1 \vee p = -1 \\ p = 1 \text{ geeft } rc_k &= \frac{y_A}{x_A} = \frac{1^2 - 5 \cdot 1 + 6}{1} = 2 \\ p = -1 \text{ geeft } rc_k &= \frac{y_A}{x_A} = \frac{(-1)^2 - 5 \cdot (-1) + 6}{-1} = -12 \end{aligned}$$

Bladzijde 96

- 9 a** $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 4$ geeft $f'(x) = x^2 - 4x + 3$
 $f'(x) = 0$ geeft $x^2 - 4x + 3 = 0$
 $(x - 1)(x - 3) = 0$
 $x = 1 \vee x = 3$

$$f(1) = 5\frac{1}{3} \text{ en } f(3) = 4$$

Dus $A(1, 5\frac{1}{3})$ en $B(3, 4)$.

- b** $OA = \sqrt{1^2 + (5\frac{1}{3})^2} \approx 5,43$

$$OB = \sqrt{3^2 + 4^2} = 5$$

- 10** $p = 2\frac{1}{3}$ geeft $y_A = 1\frac{1}{3} + 2\frac{1}{3} = 3\frac{2}{3}$ en $y_B = 2\frac{1}{3}$

$$A(1, 3\frac{2}{3}) \text{ en } B(3, 2\frac{1}{3}) \text{ geeft } AB = \sqrt{(3-1)^2 + (2\frac{1}{3} - 3\frac{2}{3})^2} \approx 2,40$$

11 $f_p(x) = p + \sin(x)$ geeft $f_p'(x) = \cos(x)$

$$f_p'(x) = 0 \text{ geeft } \cos(x) = 0$$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

Dus $A(\frac{1}{2}\pi, p+1)$ en $B(\frac{1}{2}\pi, p-1)$.

$$OA = OB \text{ geeft } OA^2 = OB^2$$

$$(\frac{1}{2}\pi)^2 + (p+1)^2 = (\frac{1}{2}\pi)^2 + (p-1)^2$$

$$\frac{1}{4}\pi^2 + p^2 + 2p + 1 = \frac{1}{4}\pi^2 + p^2 - 2p + 1$$

$$4p = 2\pi^2$$

$$p = \frac{1}{2}\pi^2$$

Bladzijde 97

12 $g_p(x) = \frac{3x^2 + 10x - 3}{x^2 + 4} - p$ geeft

$$g_p'(x) = \frac{(x^2 + 4) \cdot (6x + 10) - (3x^2 + 10x - 3) \cdot 2x}{(x^2 + 4)^2} = \frac{6x^3 + 10x^2 + 24x + 40 - 6x^3 - 20x^2 + 6x}{(x^2 + 4)^2} = \frac{-10x^2 + 30x + 40}{(x^2 + 4)^2}$$

$$g_p'(x) = 0 \text{ geeft } -10x^2 + 30x + 40 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x+1)(x-4) = 0$$

$$x = -1 \vee x = 4$$

$$g_p(-1) = -2 - p \text{ en } g_p(4) = 4\frac{1}{4} - p, \text{ dus } C(-1, -2 - p) \text{ en } D(4, 4\frac{1}{4} - p).$$

$$OC = OD \text{ geeft } OC^2 = OD^2$$

$$(-1)^2 + (-2 - p)^2 = 4^2 + (4\frac{1}{4} - p)^2$$

$$1 + 4 + 4p + p^2 = 16 + 18\frac{1}{16} - 8\frac{1}{2}p + p^2$$

$$12\frac{1}{2}p = 29\frac{1}{16}$$

$$p = 2\frac{13}{40}$$

13 $f_p(x) = -0,03x^4 + 0,08x^3 + 0,48x^2 + p$ geeft $f_p'(x) = -0,12x^3 + 0,24x^2 + 0,96x$

$$f_p'(x) = 0 \text{ geeft } -0,12x^3 + 0,24x^2 + 0,96x = 0$$

$$x^3 - 2x^2 - 8x = 0$$

$$x(x^2 - 2x - 8) = 0$$

$$x(x+2)(x-4) = 0$$

$$x = 0 \vee x = -2 \vee x = 4$$

$$f_p(-2) = 0,8 + p \text{ en } f_p(4) = 5,12 + p, \text{ dus } A(-2; 0,8 + p) \text{ en } B(4; 5,12 + p).$$

$$OB = 2OA \text{ geeft } OB^2 = 4OA^2$$

$$4^2 + (5,12 + p)^2 = 4((-2)^2 + (0,8 + p)^2)$$

$$16 + 26,2144 + 10,24p + p^2 = 4(4 + 0,64 + 1,6p + p^2)$$

$$16 + 26,2144 + 10,24p + p^2 = 16 + 2,56 + 6,4p + 4p^2$$

$$-3p^2 + 3,84p + 23,6544 = 0$$

$$p^2 - 1,28p - 7,8848 = 0$$

$$D = (-1,28)^2 - 4 \cdot 1 \cdot -7,8848 = 33,1776$$

$$p = \frac{1,28 + 5,76}{2} = 3,52 \vee p = \frac{1,28 - 5,76}{2} = -2,24$$

vold. niet

Dus $p = 3,52$.

14 a $f_1(x) = \frac{\sin(x)}{1 - \cos^2(x)} = \frac{\sin(x)}{\sin^2(x)} = \frac{1}{\sin(x)} = (\sin(x))^{-1}$ geeft $f_1'(x) = -(\sin(x))^{-2} \cdot \cos(x) = -\frac{\cos(x)}{\sin^2(x)}$

$$f_1'(x) = 0 \text{ geeft } \cos(x) = 0$$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

$$\text{Dus } x_A = \frac{1}{2}\pi \text{ en } x_B = 1\frac{1}{2}\pi.$$

$$f_1\left(\frac{1}{2}\pi\right) = \frac{1}{\sin\left(\frac{1}{2}\pi\right)} = \frac{1}{1} = 1, \text{ dus } A\left(\frac{1}{2}\pi, 1\right).$$

$$f_1\left(1\frac{1}{2}\pi\right) = \frac{1}{\sin\left(1\frac{1}{2}\pi\right)} = \frac{1}{-1} = -1, \text{ dus } B\left(1\frac{1}{2}\pi, -1\right).$$

$$AB = \sqrt{\left(1\frac{1}{2}\pi - \frac{1}{2}\pi\right)^2 + (-1 - 1)^2} \approx 3,72$$

b $f_p(x) = \frac{\sin(x)}{p - \cos^2(x)}$ geeft

$$f_p'(x) = \frac{(p - \cos^2(x)) \cdot \cos(x) - \sin(x) \cdot -2 \cos(x) \cdot -\sin(x)}{(p - \cos^2(x))^2} = \frac{p \cos(x) - \cos^3(x) - 2 \sin^2(x) \cos(x)}{(p - \cos^2(x))^2}$$

$$f_p'\left(\frac{1}{2}\pi\right) = \frac{p \cos\left(\frac{1}{2}\pi\right) - \cos^3\left(\frac{1}{2}\pi\right) - 2 \sin^2\left(\frac{1}{2}\pi\right) \cos\left(\frac{1}{2}\pi\right)}{(p - \cos^2\left(\frac{1}{2}\pi\right))^2} = \frac{p \cdot 0 - 0^3 - 2 \cdot 1^2 \cdot 0}{(p - 0^2)^2} = \frac{0}{p^2} = 0$$

Dus de raaklijn van de grafiek van f_p is horizontaal in het punt E .

c $f_p\left(\frac{1}{2}\pi\right) = \frac{1}{p - 0^2} = \frac{1}{p}$, dus $E\left(\frac{1}{2}\pi, \frac{1}{p}\right)$.

$$OC = \pi \text{ geeft } OC^2 = \pi^2$$

$$OE^2 = \left(\frac{1}{2}\pi\right)^2 + \left(\frac{1}{p}\right)^2 = \frac{1}{4}\pi^2 + \frac{1}{p^2}$$

Uit $O(0, 0)$, $C(\pi, 0)$ en $x_E = \frac{1}{2}\pi$ volgt $OE = CE$.

$$OC = OE \text{ geeft } OC^2 = OE^2$$

$$\pi^2 = \frac{1}{4}\pi^2 + \frac{1}{p^2}$$

$$\frac{1}{p^2} = \frac{3}{4}\pi^2$$

$$p^2 = \frac{4}{3\pi^2}$$

$$p = \frac{2}{3\pi}\sqrt{3} \vee p = -\frac{2}{3\pi}\sqrt{3}$$

vold. niet

d $\overrightarrow{OE} = \begin{pmatrix} \frac{1}{2}\pi \\ \frac{1}{p} \end{pmatrix}$ en $\overrightarrow{EC} = \begin{pmatrix} \frac{1}{2}\pi \\ -\frac{1}{p} \end{pmatrix}$

$$\overrightarrow{OE} \cdot \overrightarrow{EC} = 0 \text{ geeft } \begin{pmatrix} \frac{1}{2}\pi \\ \frac{1}{p} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2}\pi \\ -\frac{1}{p} \end{pmatrix} = 0$$

$$\frac{1}{4}\pi^2 - \frac{1}{p^2} = 0$$

$$\frac{1}{4}\pi^2 = \frac{1}{p^2}$$

$$p^2 = \frac{4}{\pi^2}$$

$$p = \frac{2}{\pi} \vee p = -\frac{2}{\pi}$$

vold. niet

Alternatieve uitwerking

Uit $OE = CE$ (zie c) en $\angle(OE, CE) = 90^\circ$ volgt $\angle COE = \angle OCE = 45^\circ$ (gelijkbenige rechthoekige driehoek), dus $rc_{OE} = 1$. Hieruit volgt $x_E = y_E$

$$\frac{1}{2}\pi = \frac{1}{p}$$

$$p = \frac{2}{\pi}$$

15.2 Optimaliseringsproblemen**Bladzijde 99**

15 a $L = y_A - y_B = f(p) - g(p) = \sqrt{2p} - \frac{1}{2}p$

b Voer in $y_1 = \sqrt{2x} - \frac{1}{2}x$.

De optie maximum geeft $x = 2$.

Voor $p = 2$ is de lengte van lijnstuk AB maximaal.

Bladzijde 100

16 a $L = 2 \cos^2(p) - \sin(2p) + 1 = \cos(2p) + 1 - \sin(2p) + 1 = 2 + \cos(2p) - \sin(2p)$

$p = \frac{7}{8}\pi$ geeft $L = 2 + \cos(\frac{3}{4}\pi) - \sin(\frac{3}{4}\pi) = 2 + \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} = 2 + \sqrt{2}$

Het maximum van L is $2 + \sqrt{2}$.

b Manier 1: $\sin(2p) = -\cos(2p)$
 $\sin(2p) = \cos(2p + \pi)$
 $\sin(2p) = \sin(2p + \frac{1}{2}\pi)$
 enzovoort

Manier 2: $\sin(2p) = -\cos(2p)$
 $\sin(2p) = \cos(2p + \pi)$
 $\cos(2p - \frac{1}{2}\pi) = \cos(2p + \pi)$
 enzovoort

Bladzijde 101

17 a $L = f(p) - g(p) = \sqrt{6p + 12} - (p + 2) = \sqrt{6p + 12} - p - 2$

b $\frac{dL}{dp} = \frac{1}{2\sqrt{6p+12}} \cdot 6 - 1 = \frac{3}{\sqrt{6p+12}} - 1$

$\frac{dL}{dp} = 0$ geeft $\frac{3}{\sqrt{6p+12}} - 1 = 0$

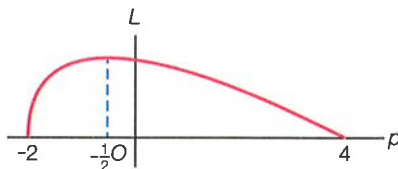
$$\frac{3}{\sqrt{6p+12}} = 1$$

$$\sqrt{6p+12} = 3$$

$$6p+12 = 9$$

$$6p = -3$$

$$p = -\frac{1}{2}$$



De maximale waarde van L is $\sqrt{6 \cdot -\frac{1}{2} + 12} - \frac{1}{2} - 2 = 1\frac{1}{2}$.

18 $L = AB = f(p) - g(p) = \frac{1}{2} \sin(2p) - (\cos(p) - 1\frac{1}{2}) = \frac{1}{2} \sin(2p) - \cos(p) + 1\frac{1}{2}$

$$\frac{dL}{dp} = \frac{1}{2} \cdot \cos(2p) \cdot 2 + \sin(p) = \cos(2p) + \sin(p)$$

$$\frac{dL}{dp} = 0 \text{ geeft } \cos(2p) + \sin(p) = 0$$

$$\cos(2p) = -\sin(p)$$

$$\cos(2p) = \sin(p + \pi)$$

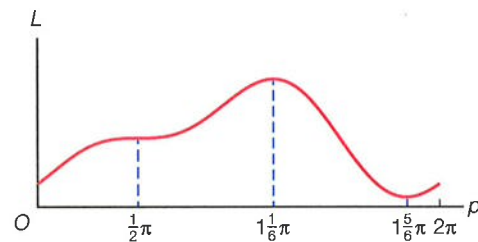
$$\cos(2p) = \cos(p + \frac{1}{2}\pi)$$

$$2p = p + \frac{1}{2}\pi + k \cdot 2\pi \vee 2p = -p - \frac{1}{2}\pi + k \cdot 2\pi$$

$$p = \frac{1}{2}\pi + k \cdot 2\pi \vee 3p = -\frac{1}{2}\pi + k \cdot 2\pi$$

$$p = \frac{1}{2}\pi + k \cdot 2\pi \vee p = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

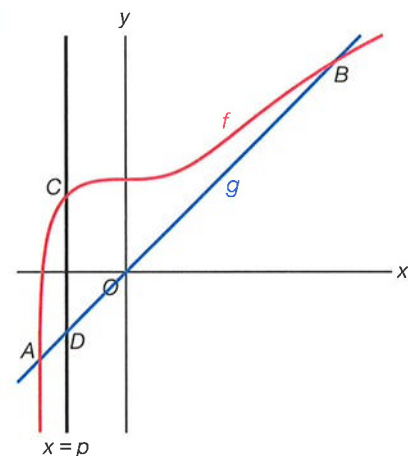
$$p \text{ in } [0, 2\pi] \text{ geeft } p = \frac{1}{2}\pi \vee p = 1\frac{1}{6}\pi \vee p = 1\frac{5}{6}\pi$$



De maximale lengte van AB is

$$\frac{1}{2} \sin(2 \cdot 1\frac{1}{6}\pi) - \cos(1\frac{1}{6}\pi) + 1\frac{1}{2} = \frac{1}{2} \sin(2\frac{1}{3}\pi) - \cos(1\frac{1}{6}\pi) + 1\frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} + 1\frac{1}{2} = \frac{3}{4}\sqrt{3} + 1\frac{1}{2}.$$

19

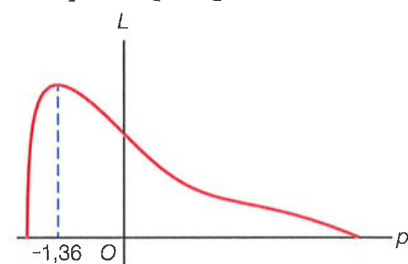


$$L = CD = f(p) - g(p) = \ln(p^3 + 8) - p$$

$$\frac{dL}{dp} = \frac{1}{p^3 + 8} \cdot 3p^2 - 1 = \frac{3p^2}{p^3 + 8} - 1$$

$$\text{Voer in } y_1 = \frac{3x^2}{x^3 + 8} - 1.$$

De optie nulpunt geeft $x \approx -1,36$.



Dus de lengte van lijnstuk CD is maximaal voor $p \approx -1,36$.

- 20 a** Voor f geldt $y = e^{\frac{1}{2}x}$, dus voor f^{inv} geldt $x = e^{\frac{1}{2}y}$.

$$x = e^{\frac{1}{2}y} \text{ geeft } e^{\frac{1}{2}y} = x$$

$$\frac{1}{2}y = \ln(x)$$

$$y = 2 \ln(x)$$

- b** $y = 2 \ln(x) \xrightarrow{\text{verm. } y=3} y = 2 \ln(\frac{1}{3}x)$, dus $g(x) = 2 \ln(\frac{1}{3}x)$.

$$L = AB = f(p) - g(p) = e^{\frac{1}{2}p} - 2 \ln(\frac{1}{3}p)$$

$$\text{Voer in } y_1 = e^{\frac{1}{2}x} - 2 \ln(\frac{1}{3}x).$$

De optie minimum geeft $x \approx 1,71$ en $y \approx 3,48$.

Dus de minimale lengte van het lijnstuk AB is ongeveer 3,48.

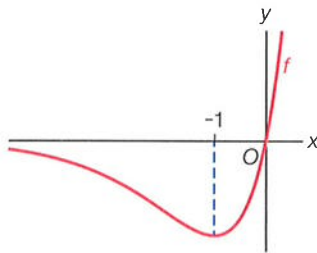
Bladzijde 102

- 21 a** $f(x) = 5x e^x$ geeft $f'(x) = 5 \cdot e^x + 5x \cdot e^x = (5x + 5) e^x$

$$f'(x) = 0 \text{ geeft } 5x + 5 = 0$$

$$5x = -5$$

$$x = -1$$



$$\text{min. is } f(-1) = 5 \cdot -1 \cdot e^{-1} = \frac{-5}{e}$$

$$\text{Dus } B_f = \left[-\frac{5}{e}, \rightarrow \right).$$

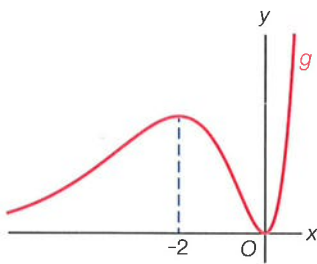
- b** $g(x) = 5x^2 e^x$ geeft $g'(x) = 10x \cdot e^x + 5x^2 \cdot e^x = (5x^2 + 10x) e^x$

$$g'(x) = 0 \text{ geeft } (5x^2 + 10x) e^x = 0$$

$$5x^2 + 10x = 0$$

$$5x(x + 2) = 0$$

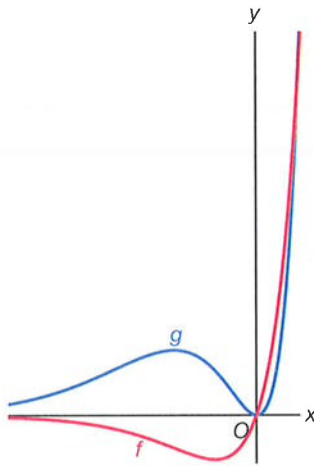
$$x = 0 \vee x = -2$$



$$\text{max. is } g(-2) = 5 \cdot (-2)^2 \cdot e^{-2} = \frac{20}{e^2}$$

$$\text{min. is } g(0) = 5 \cdot 0^2 \cdot e^0 = 0$$

c



$$L = AB = g(p) - f(p) = 5p^2 e^p - 5p e^p = (5p^2 - 5p) e^p$$

$$\frac{dL}{dp} = (10p - 5) \cdot e^p + (5p^2 - 5p) \cdot e^p = (5p^2 + 5p - 5) e^p$$

$$\frac{dL}{dp} = 0 \text{ geeft } (5p^2 + 5p - 5) e^p = 0$$

$$5p^2 + 5p - 5 = 0$$

$$p^2 + p - 1 = 0$$

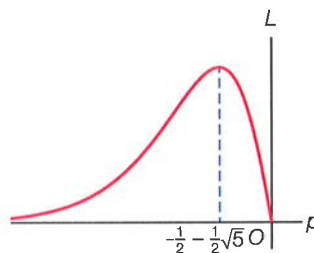
$$\left(p + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0$$

$$\left(p + \frac{1}{2}\right)^2 = \frac{5}{4}$$

$$p + \frac{1}{2} = \frac{1}{2}\sqrt{5} \vee p + \frac{1}{2} = -\frac{1}{2}\sqrt{5}$$

$$p = -\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee p = -\frac{1}{2} - \frac{1}{2}\sqrt{5}$$

vold. niet



Dus de lengte van het lijnstuk AB is maximaal voor $p = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$.

d $L = CD = f(q) - g(q) = 5q e^q - 5q^2 e^q$

Voer in $y_1 = 5x e^x - 5x^2 e^x$.

De optie maximum geeft $x \approx 0,618$ en $y \approx 2,190$.

De maximale lengte van het lijnstuk CD is ongeveer 2,190.

22 a De kettingboog door $(0, 10)$ geeft $\frac{1}{2}a(e^0 + e^0) + b = 10$

$$\frac{1}{2}a(1 + 1) + b = 10$$

$$a + b = 10$$

$$b = 10 - a$$

De kettingboog door $(10, 0)$ geeft $\frac{1}{2}a(e^{\frac{10}{a}} + e^{-\frac{10}{a}}) + b = 0$

$$b = -\frac{1}{2}a(e^{\frac{10}{a}} + e^{-\frac{10}{a}})$$

Voer in $y_1 = 10 - x$ en $y_2 = -\frac{1}{2}x(e^{\frac{10}{x}} + e^{-\frac{10}{x}})$.

De optie snijpunt geeft $x = -6,1875...$ en $y = 16,1875...$

Dus $a \approx -6,188$ en $b \approx 16,188$.

$$\text{b } \left. \begin{array}{l} y_p = mx^2 + n \\ \text{door } (0, 10) \end{array} \right\} n = 10$$

$$\left. \begin{array}{l} y_p = mx^2 + 10 \\ \text{door } (10, 0) \end{array} \right\} \begin{array}{l} m \cdot 10^2 + 10 = 0 \\ 100m = -10 \\ m = -0,1 \end{array}$$

Dus $y_p = -0,1x^2 + 10$.

Het verschil tussen y_k en y_p is

$$|y_k - y_p| = |-3,0937... \cdot (e^{\frac{x}{-6,1875...}} + e^{-\frac{x}{-6,1875...}}) + 16,1875... - (-0,1x^2 + 10)|.$$

$$\text{Voer in } y_1 = |-3,0937... \cdot (e^{\frac{x}{-6,1875...}} + e^{-\frac{x}{-6,1875...}}) + 16,1875... - (-0,1x^2 + 10)|.$$

De optie maximum geeft $x = 7,1448...$ en $y = 0,5005...$

Dus het maximale verschil tussen y_k en y_p is ongeveer 50 cm.

$$23 \text{ a } y_Q = f(2) = \sqrt{5-4} = 1$$

$$OQ = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\text{b } y_Q = f(p) = \sqrt{5-2p}$$

$$L = OQ = \sqrt{x_Q^2 + y_Q^2} = \sqrt{q^2 + (\sqrt{5-2q})^2} = \sqrt{q^2 - 2q + 5}$$

$$\text{c } \text{Voer in } y_1 = \sqrt{x^2 - 2x + 5}.$$

De optie minimum geeft $x = 1$ en $y = 2$.

De minimale lengte van het lijnstuk OQ is 2.

Bladzijde 103

$$24 \text{ a } A = p\sqrt{5-2p} \text{ geeft}$$

$$\frac{dA}{dp} = 1 \cdot \sqrt{5-2p} + p \cdot \frac{1}{2\sqrt{5-2p}} \cdot -2 = \sqrt{5-2p} - \frac{p}{\sqrt{5-2p}} = \frac{5-2p}{\sqrt{5-2p}} - \frac{p}{\sqrt{5-2p}} = \frac{5-3p}{\sqrt{5-2p}}$$

$$\text{b } \frac{5}{3}\sqrt{5-2} \cdot \frac{5}{3} = \frac{5}{3}\sqrt{5-\frac{10}{3}} = \frac{5}{3}\sqrt{\frac{5}{3}} = \frac{5}{3} \cdot \frac{\sqrt{5}}{\sqrt{3}} = \frac{5}{3} \cdot \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{3} \cdot \frac{\sqrt{15}}{3} = \frac{5}{9}\sqrt{15}$$

$$25 \text{ a } OQ = x_p = p \text{ en } PQ = y_p = f(p) = \sqrt{3-p}$$

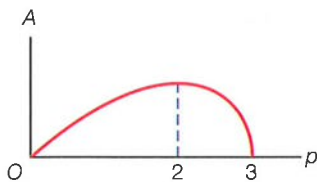
$$A = O(\triangle OPQ) = \frac{1}{2} \cdot OQ \cdot PQ = \frac{1}{2} \cdot p \cdot \sqrt{3-p} = \frac{1}{2}p\sqrt{3-p}$$

$$\text{b } \frac{dA}{dp} = \frac{1}{2} \cdot \sqrt{3-p} + \frac{1}{2}p \cdot \frac{1}{2\sqrt{3-p}} \cdot -1 = \frac{\sqrt{3-p}}{2} - \frac{p}{4\sqrt{3-p}} = \frac{2(3-p)-p}{4\sqrt{3-p}} = \frac{6-3p}{4\sqrt{3-p}}$$

$$\text{c } \frac{dA}{dp} = 0 \text{ geeft } 6-3p = 0$$

$$-3p = -6$$

$$p = 2$$



De maximale oppervlakte van driehoek OPQ is $\frac{1}{2} \cdot 2 \cdot \sqrt{3-2} = 1$.

$$\text{d } OQ = x_p = p \text{ en } PQ = y_p = f(p) = \sqrt{3-p}$$

$$\text{In } \triangle OPQ \text{ is } OP^2 = OQ^2 + PQ^2$$

$$OP^2 = p^2 + (\sqrt{3-p})^2$$

$$OP^2 = p^2 + 3 - p$$

$$OP^2 = p^2 - p + 3$$

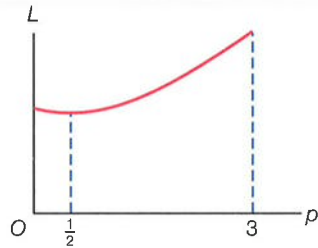
$$L = OP = \sqrt{p^2 - p + 3}$$

$$\text{e } L = \sqrt{p^2 - p + 3} \text{ geeft } \frac{dL}{dp} = \frac{1}{2\sqrt{p^2 - p + 3}} \cdot (2p - 1) = \frac{2p - 1}{2\sqrt{p^2 - p + 3}}$$

$$\frac{dL}{dp} = 0 \text{ geeft } 2p - 1 = 0$$

$$2p = 1$$

$$p = \frac{1}{2}$$



De minimale lengte van OP is $\sqrt{(\frac{1}{2})^2 - \frac{1}{2} + 3} = \sqrt{2\frac{3}{4}} = \sqrt{\frac{11}{4}} = \frac{1}{2}\sqrt{11}$.

Bladzijde 104

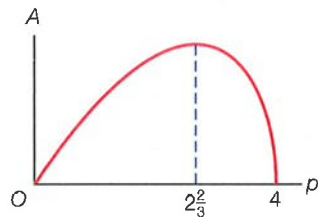
26 a $A = O(\triangle OSP) = \frac{1}{2} \cdot OS \cdot y_P = \frac{1}{2} \cdot 4 \cdot p\sqrt{8-2p} = 2p\sqrt{8-2p}$

$$\frac{dA}{dp} = 2 \cdot \sqrt{8-2p} + 2p \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = 2\sqrt{8-2p} - \frac{2p}{\sqrt{8-2p}} = \frac{2(8-2p) - 2p}{\sqrt{8-2p}} = \frac{16-6p}{\sqrt{8-2p}}$$

$$\frac{dA}{dp} = 0 \text{ geeft } 16 - 6p = 0$$

$$-6p = -16$$

$$p = 2\frac{2}{3}$$



De maximale oppervlakte van driehoek OSP is

$$2 \cdot 2\frac{2}{3}\sqrt{8-2 \cdot 2\frac{2}{3}} = 5\frac{1}{3}\sqrt{2\frac{2}{3}} = 5\frac{1}{3}\sqrt{\frac{8}{3}} = 5\frac{1}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} = 5\frac{1}{3} \cdot \frac{2\sqrt{6}}{3} = 3\frac{5}{9}\sqrt{6}.$$

Opmerking

Je krijgt de maximale oppervlakte als P de top is. Je kunt dus ook eerst y_{top} berekenen.

b $QS = x_S - x_Q = 4 - p$ en $PQ = y_P = f(p) = p\sqrt{8-2p}$

$$O = O(\triangle QSP) = \frac{1}{2} \cdot QS \cdot PQ = \frac{1}{2} \cdot (4-p) \cdot p\sqrt{8-2p} = (2p - \frac{1}{2}p^2)\sqrt{8-2p}$$

c $\frac{dO}{dp} = (2-p) \cdot \sqrt{8-2p} + (2p - \frac{1}{2}p^2) \cdot \frac{1}{2\sqrt{8-2p}} \cdot -2 = (2-p)\sqrt{8-2p} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}}$

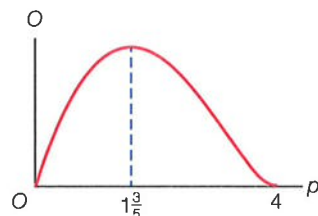
$$= \frac{(2-p)(8-2p)}{\sqrt{8-2p}} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}} = \frac{16-4p-8p+2p^2-2p+\frac{1}{2}p^2}{\sqrt{8-2p}} = \frac{2\frac{1}{2}p^2-14p+16}{\sqrt{8-2p}}$$

$$= \frac{5p^2-28p+32}{2\sqrt{8-2p}}$$

$$\frac{dO}{dp} = 0 \text{ geeft } 5p^2 - 28p + 32 = 0$$

$$D = (-28)^2 - 4 \cdot 5 \cdot 32 = 144$$

$$p = \frac{28 \pm 12}{10} = 4 \vee p = \frac{28-12}{10} = 1\frac{3}{5}$$



De maximale oppervlakte van driehoek QSP is $(2 \cdot 1\frac{3}{5} - \frac{1}{2} \cdot (1\frac{3}{5})^2)\sqrt{8-2 \cdot 1\frac{3}{5}} \approx 4,21$.

- 27** Stel $x_P = p$.
 Dan is $y_P = p\sqrt{p}$.

$$A(4, 0) \text{ en } P(p, p\sqrt{p}) \text{ geeft } AP = \sqrt{(p-4)^2 + (p\sqrt{p}-0)^2} = \sqrt{p^2 - 8p + 16 + p^3}$$

$$\text{Voer in } y_1 = \sqrt{x^2 - 8x + 16 + x^3}.$$

De optie minimum geeft $x \approx 1,33$ en $y \approx 3,08$.

Dus deze minimale lengte is ongeveer 3,08.

- 28 a** Stel $x_P = p$.

$$PQ = x_P - x_Q = p - -p = 2p \text{ en } y_P = 3 - \frac{1}{2}p^2$$

$$A = O(\triangle OPQ) = \frac{1}{2} \cdot PQ \cdot y_P = \frac{1}{2} \cdot 2p \cdot (3 - \frac{1}{2}p^2) = 3p - \frac{1}{2}p^3$$

$$\frac{dA}{dp} = 3 - \frac{3}{2}p^2$$

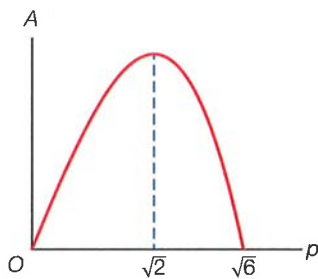
$$\frac{dA}{dp} = 0 \text{ geeft } 3 - \frac{3}{2}p^2 = 0$$

$$-\frac{3}{2}p^2 = -3$$

$$p^2 = 2$$

$$p = \sqrt{2} \vee p = -\sqrt{2}$$

vold. niet



Dus de maximale oppervlakte van driehoek OPQ is $3\sqrt{2} - \frac{1}{2}(\sqrt{2})^3 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$.

- b** $OP^2 = x_P^2 + y_P^2 = p^2 + (3 - \frac{1}{2}p^2)^2 = p^2 + 9 - 3p^2 + \frac{1}{4}p^4 = \frac{1}{4}p^4 - 2p^2 + 9$

Stel de lengte van OP is L .

$$L = \sqrt{\frac{1}{4}p^4 - 2p^2 + 9} \text{ geeft } \frac{dL}{dp} = \frac{1}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}} \cdot (p^3 - 4p) = \frac{p^3 - 4p}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}}$$

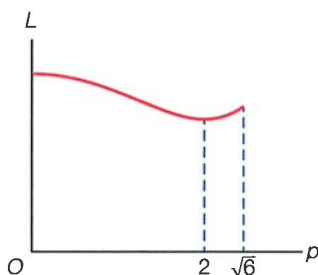
$$\frac{dL}{dp} = 0 \text{ geeft } p^3 - 4p = 0$$

$$p(p^2 - 4) = 0$$

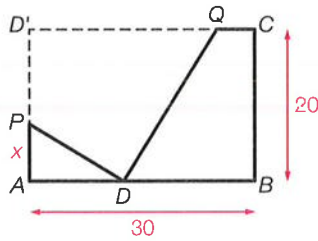
$$p = 0 \vee p^2 = 4$$

$$p = 0 \vee p = 2 \vee p = -2$$

vold. niet



De minimale lengte van het lijnstuk OP is $\sqrt{\frac{1}{4} \cdot 2^4 - 2 \cdot 2^2 + 9} = \sqrt{5}$.

Bladzijde 105**29** Stel $AP = x$.

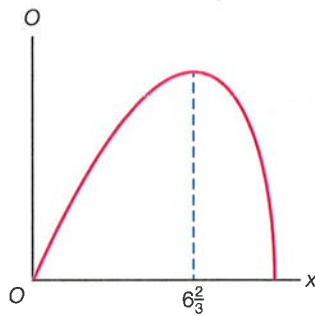
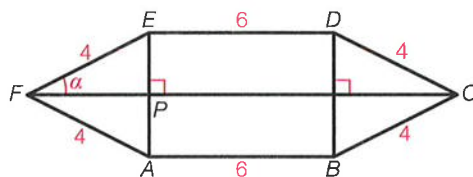
$$\left. \begin{array}{l} AP + PD' = 20 \\ AP = x \\ PD' = PD \end{array} \right\} x + PD = 20, \text{ dus } PD = 20 - x$$

$$\begin{aligned} \text{In } \triangle ADP \text{ is } AP^2 + AD^2 &= PD^2 \\ x^2 + AD^2 &= (20 - x)^2 \\ x^2 + AD^2 &= 400 - 40x + x^2 \\ AD^2 &= 400 - 40x \\ AD &= \sqrt{400 - 40x} \end{aligned}$$

$$O = O(\triangle ADP) = \frac{1}{2} \cdot AD \cdot AP = \frac{1}{2} \cdot \sqrt{400 - 40x} \cdot x = \frac{1}{2} x \sqrt{400 - 40x}$$

$$\frac{dO}{dx} = \frac{1}{2} \cdot \sqrt{400 - 40x} + \frac{1}{2} x \cdot \frac{1}{2\sqrt{400 - 40x}} \cdot (-40) = \frac{\frac{1}{2}(400 - 40x)}{\sqrt{400 - 40x}} - \frac{10x}{\sqrt{400 - 40x}} = \frac{200 - 20x - 10x}{\sqrt{400 - 40x}} = \frac{200 - 30x}{\sqrt{400 - 40x}}$$

$$\begin{aligned} \frac{dO}{dx} = 0 \text{ geeft } 200 - 30x &= 0 \\ -30x &= -200 \\ x &= 6\frac{2}{3} \end{aligned}$$

Dus O is maximaal voor $x = 6\frac{2}{3}$.De maximale oppervlakte is $\frac{1}{2} \cdot 6\frac{2}{3} \cdot \sqrt{400 - 40 \cdot 6\frac{2}{3}} \approx 38,49 \text{ cm}^2$.**30 a**

$$\text{In } \triangle FPE \text{ is } \sin(\alpha) = \frac{EP}{4}, \text{ dus } EP = 4 \sin(\alpha).$$

$$AE = 2EP = 8 \sin(\alpha)$$

$$\text{b In } \triangle FPE \text{ is } \cos(\alpha) = \frac{FP}{4}, \text{ dus } FP = 4 \cos(\alpha).$$

$$O(\triangle AEF) = \frac{1}{2} \cdot FP \cdot AE = \frac{1}{2} \cdot 4 \cos(\alpha) \cdot 8 \sin(\alpha) = 16 \sin(\alpha) \cos(\alpha)$$

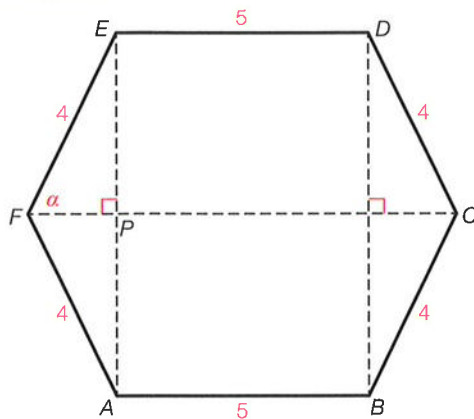
$$\text{c } O(ABDE) = AB \cdot AE = 6 \cdot 8 \sin(\alpha) = 48 \sin(\alpha)$$

$$\text{d } O(ABCDEF) = 2 \cdot O(\triangle AEF) + O(ABDE) = 32 \sin(\alpha) \cos(\alpha) + 48 \sin(\alpha) = 16 \sin(2\alpha) + 48 \sin(\alpha)$$

Bladzijde 106

31

a



$$\cos(\alpha) = \frac{FP}{4} \text{ geeft } FP = 4 \cos(\alpha)$$

$$\sin(\alpha) = \frac{EP}{4} \text{ geeft } EP = 4 \sin(\alpha)$$

$$AE = 2EP = 8 \sin(\alpha)$$

$$A = O(ABCDEF) = 2 \cdot O(\triangle AEF) + O(ABDE) = 2 \cdot \frac{1}{2} \cdot 8 \sin(\alpha) \cdot 4 \cos(\alpha) + 5 \cdot 8 \sin(\alpha) \\ = 32 \sin(\alpha) \cos(\alpha) + 40 \sin(\alpha) = 16 \sin(2\alpha) + 40 \sin(\alpha)$$

$$\text{b } \frac{dA}{d\alpha} = 16 \cdot \cos(2\alpha) \cdot 2 + 40 \cos(\alpha) = 32 \cos(2\alpha) + 40 \cos(\alpha)$$

$$\frac{dA}{d\alpha} = 0 \text{ geeft } 32 \cos(2\alpha) + 40 \cos(\alpha) = 0 \\ 32(2 \cos^2(\alpha) - 1) + 40 \cos(\alpha) = 0$$

$$64 \cos^2(\alpha) - 32 + 40 \cos(\alpha) = 0$$

$$8 \cos^2(\alpha) + 5 \cos(\alpha) - 4 = 0$$

$$\text{Stel } \cos(\alpha) = u.$$

$$8u^2 + 5u - 4 = 0$$

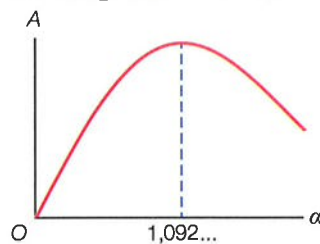
$$D = 5^2 - 4 \cdot 8 \cdot -4 = 153$$

$$u = \frac{-5 + \sqrt{153}}{16} = 0,460... \vee u = \frac{-5 - \sqrt{153}}{16} = -1,085...$$

$$\cos(\alpha) = 0,460... \vee \cos(\alpha) = -1,085...$$

$$\alpha = 1,092... + k \cdot 2\pi \vee \alpha = -1,092... + k \cdot 2\pi$$

$$\alpha \text{ in } [0, \frac{1}{2}\pi] \text{ geeft } \alpha = 1,092...$$

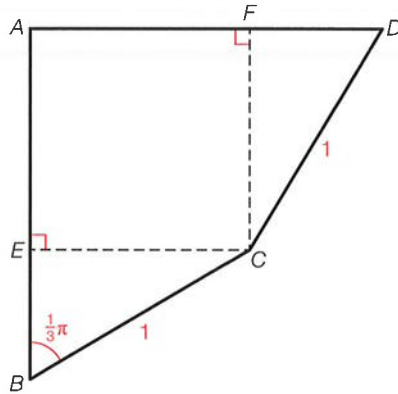


$$\text{De oppervlakte is maximaal bij } \alpha = 1,092... \cdot \frac{180^\circ}{\pi} \approx 63^\circ.$$

Bladzijde 107

- 32 a Bij $\angle B = \frac{1}{2}\pi$ is $O(ABCD) = 1^2 = 1$.

Zie voor $\angle B = \frac{1}{3}\pi$ de figuur.



In $\triangle BCE$ is $CE = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$ en $DE = \cos(\frac{1}{3}\pi) = \frac{1}{2}$.

$$O(ABCD) = 2 \cdot O(\triangle BCE) + O(AECF) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} \cdot \frac{1}{2} + (\frac{1}{2}\sqrt{3})^2 \approx 1,18$$

- b $CE = \sin(x)$ en $DE = \cos(x)$

$$O(ABCD) = 2 \cdot O(\triangle BCE) + O(AECF) = 2 \cdot \frac{1}{2} \cdot \sin(x) \cdot \cos(x) + \sin^2(x) = \sin^2(x) + \sin(x)\cos(x)$$

- c $A = O(ABCD) = \sin^2(x) + \sin(x)\cos(x) = \sin^2(x) + \frac{1}{2}\sin(2x)$ geeft

$$\frac{dA}{dx} = 2\sin(x) \cdot \cos(x) + \frac{1}{2}\cos(2x) \cdot 2 = \sin(2x) + \cos(2x)$$

$$\frac{dA}{dx} = 0 \text{ geeft } \sin(2x) + \cos(2x) = 0$$

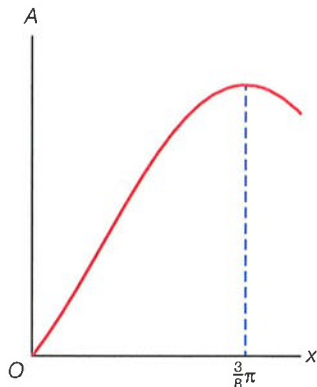
$$\sin(2x) = -\cos(2x)$$

$$\tan(2x) = -1$$

$$2x = -\frac{1}{4}\pi + k \cdot \pi$$

$$x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$0 < x < \frac{1}{2}\pi \text{ geeft } x = \frac{3}{8}\pi$$



Dus de oppervlakte van de vierhoek is maximaal voor $x = \frac{3}{8}\pi$.

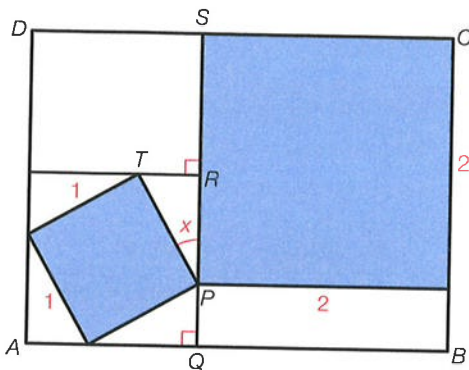
$$A = \sin^2(x) + \frac{1}{2}\sin(2x) = \frac{1}{2} - \frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x)$$

De maximale oppervlakte is

$$\frac{1}{2} - \frac{1}{2}\cos(\frac{3}{4}\pi) + \frac{1}{2}\sin(\frac{3}{4}\pi) = \frac{1}{2} - \frac{1}{2} \cdot -\frac{1}{2}\sqrt{2} + \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2} + \frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{2} = \frac{1}{2} + \frac{1}{2}\sqrt{2} \text{ m}^2.$$

33

a



In $\triangle PRT$ is $PR = \cos(x)$ en $RT = \sin(x)$.

$QP = RT = \sin(x)$, dus $QS = \sin(x) + 2$ en $O(BCSQ) = BQ \cdot QS = 2 \cdot (\sin(x) + 2) = 2 \sin(x) + 4$.

$AQ = QR = \sin(x) + \cos(x)$ geeft

$O(AQSD) = AQ \cdot QS = (\sin(x) + \cos(x))(\sin(x) + 2) = \sin^2(x) + 2 \sin(x) + \sin(x) \cos(x) + 2 \cos(x)$

$O = O(BCSQ) + O(AQSD) = 2 \sin(x) + 4 + \sin^2(x) + 2 \sin(x) + \sin(x) \cos(x) + 2 \cos(x)$
 $= \sin^2(x) + \sin(x) \cos(x) + 4 \sin(x) + 2 \cos(x) + 4$

- b Voer in $y_1 = \sin^2(x) + \sin(x) \cos(x) + 4 \sin(x) + 2 \cos(x) + 4$ en stel de GR in op graden.
 De optie maximum geeft $x \approx 65$.

Dus de oppervlakte van rechthoek $ABCD$ is maximaal voor $x \approx 65^\circ$.

Bladzijde 108

34

- a $AB = 1$, dus in $\triangle ABP$ is $AP = \cos(\alpha)$ en $BP = \sin(\alpha)$.

$\angle OAC = 180^\circ - 90^\circ - \alpha = 90^\circ - \alpha$ (gestrekte hoek) geeft

$\angle OCA = 180^\circ - 90^\circ - (90^\circ - \alpha) = \alpha$ (hoekensom driehoek)

$AC = 1$, dus in $\triangle OAC$ is $OC = \cos(\alpha)$ en $OA = \sin(\alpha)$.

$O(OPQC) = OP \cdot OC = (\sin(\alpha) + \cos(\alpha)) \cdot \cos(\alpha) = \sin(\alpha) \cos(\alpha) + \cos^2(\alpha) = \frac{1}{2} \sin(2\alpha) + \cos^2(\alpha)$

- b $V = O(OPQC) = \frac{1}{2} \sin(2\alpha) + \cos^2(\alpha)$ geeft

$$\frac{dV}{d\alpha} = \frac{1}{2} \cos(2\alpha) \cdot 2 + 2 \cos(\alpha) \cdot -\sin(\alpha) = \cos(2\alpha) - 2 \sin(\alpha) \cos(\alpha)$$

$$\frac{dV}{d\alpha} = 0 \text{ geeft } \cos(2\alpha) - 2 \sin(\alpha) \cos(\alpha) = 0$$

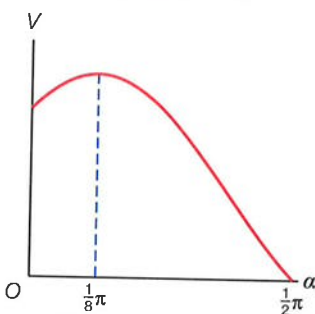
$$-\sin(2\alpha) = -\cos(2\alpha)$$

$$\tan(2\alpha) = 1$$

$$2\alpha = \frac{1}{4}\pi + k \cdot \pi$$

$$\alpha = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$0 \leq \alpha \leq \frac{1}{2}\pi \text{ geeft } \alpha = \frac{1}{8}\pi$$



Dus de oppervlakte van rechthoek $OPQC$ is maximaal voor $\alpha = \frac{1}{8}\pi$.

35

- a $MN = 7 + 5 = 12$

In $\triangle MNP$ is $\cos(x) = \frac{MP}{12}$, dus $MP = 12 \cos(x)$ en $\sin(x) = \frac{NP}{12}$, dus $NP = 12 \sin(x)$.

$$AP = AM + MP = 7 + 12 \cos(x)$$

$$BP = BN + NP = 5 + 12 \sin(x)$$

$$\begin{aligned} AB^2 &= AP^2 + BP^2 = (7 + 12 \cos(x))^2 + (5 + 12 \sin(x))^2 \\ &= 49 + 168 \cos(x) + 144 \cos^2(x) + 25 + 120 \sin(x) + 144 \sin^2(x) \\ &= 120 \sin(x) + 168 \cos(x) + 144(\sin^2(x) + \cos^2(x)) + 49 + 25 \\ &= 120 \sin(x) + 168 \cos(x) + 144 + 49 + 25 \\ &= 120 \sin(x) + 168 \cos(x) + 218 \end{aligned}$$

$$L = AB = \sqrt{120 \sin(x) + 168 \cos(x) + 218}$$

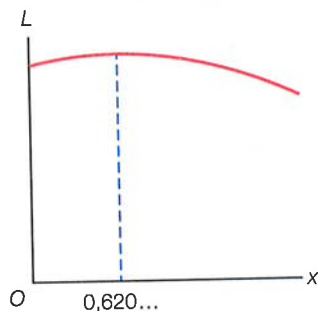
- b** $L = 20$ geeft $\sqrt{120 \sin(x) + 168 \cos(x) + 218} = 20$ oftewel $120 \sin(x) + 168 \cos(x) + 218 = 400$
 Voer in $y_1 = 120 \sin(x) + 168 \cos(x) + 218$ en $y_2 = 400$.
 De optie snijpunt geeft $x = 0,128\dots$ en $x = 1,111\dots$
 Dus $\angle PMN = 0,128\dots \cdot \frac{180^\circ}{\pi} \approx 7^\circ$ of $\angle PMN = 1,111\dots \cdot \frac{180^\circ}{\pi} \approx 64^\circ$.

- c** $L = \sqrt{120 \sin(x) + 168 \cos(x) + 218}$ geeft

$$\frac{dL}{dx} = \frac{1}{2\sqrt{120 \sin(x) + 168 \cos(x) + 218}} \cdot (120 \cos(x) - 168 \sin(x)) = \frac{60 \cos(x) - 84 \sin(x)}{\sqrt{120 \sin(x) + 168 \cos(x) + 218}}$$

Voer in $y_1 = \frac{60 \cos(x) - 84 \sin(x)}{\sqrt{120 \sin(x) + 168 \cos(x) + 218}}$.

De optie nulpunt geeft $x = 0,620\dots$

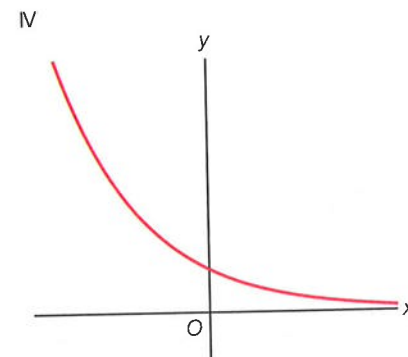
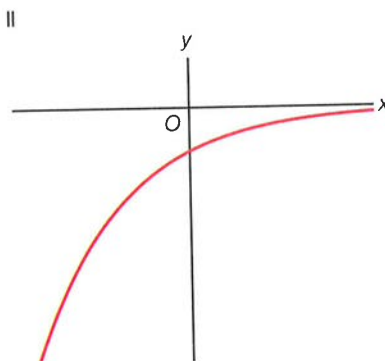
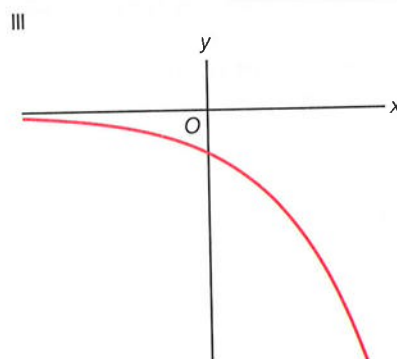
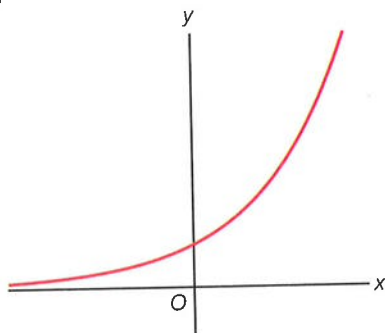


$x = 0,620\dots$ geeft $L = \sqrt{120 \sin(0,620\dots) + 168 \cos(0,620\dots) + 218} = 20,60\dots$
 De maximale afstand tussen A en B is ongeveer 206 cm.

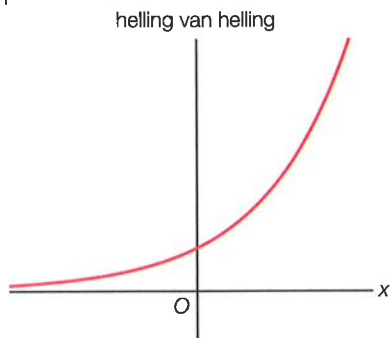
15.3 Hellingen en buigpunten

Bladzijde 110

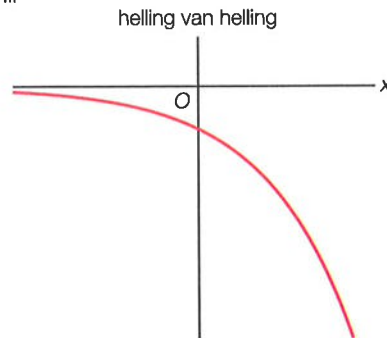
36 a i



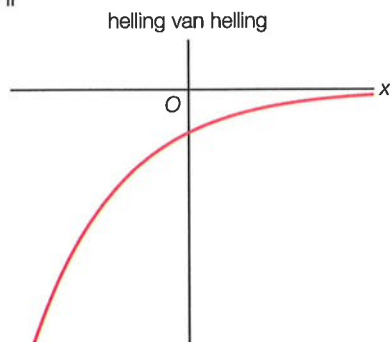
b I



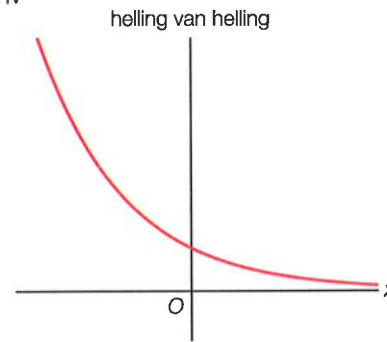
III



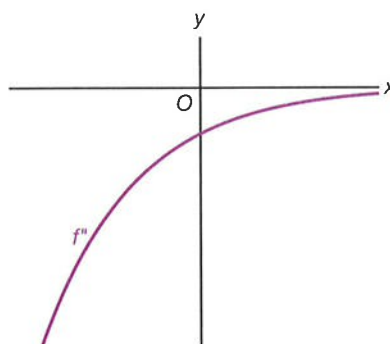
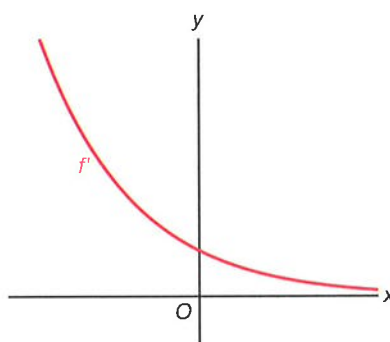
II



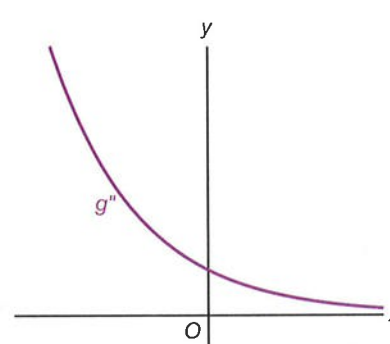
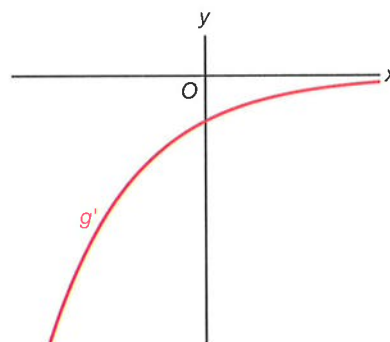
IV



37 a



b



- 38** $f(x) = x + \frac{1}{2}e^x$ geeft $f'(x) = 1 + \frac{1}{2}e^x > 0$ voor elke x , dus f is stijgend.
 $f'(x) = 1 + \frac{1}{2}e^x$ geeft $f''(x) = \frac{1}{2}e^x > 0$ voor elke x , dus f' is stijgend oftewel f is toenemend stijgend.

Bladzijde 112

- 39** $f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}$ geeft $f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2\sqrt{x}}$ en $f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4x\sqrt{x}}$

Omdat voor elke $x > 0$ geldt $x^{-\frac{1}{2}} > 0$ en $x^{-\frac{3}{2}} > 0$, is $f'(x) > 0$ en $f''(x) < 0$.
 Dus f is een afnemend stijgende functie.

40 $T = 20 + 80e^{-0,2t}$ geeft $\frac{dT}{dt} = 80e^{-0,2t} \cdot -0,2 = -16e^{-0,2t}$

$$\frac{dT}{dt} = -16e^{-0,2t} \text{ geeft } \frac{d}{dt} \left(\frac{dT}{dt} \right) = -16e^{-0,2t} \cdot -0,2 = 3,2e^{-0,2t}$$

Omdat voor elke t geldt $e^{-0,2t} > 0$, is $\frac{dT}{dt} < 0$ en $\frac{d}{dt} \left(\frac{dT}{dt} \right) > 0$.

Dus T is een afnemend dalende functie van t .

Het afkoelingsproces verloopt dus steeds langzamer.

41 $f(x) = (x^2 - 3)(x^2 - 5) = x^4 - 8x^2 + 15$

$$f'(x) = 4x^3 - 16x$$

$$f''(x) = 12x^2 - 16$$

$$f'(1) = 4 - 16 = -12 < 0$$

$$f''(1) = 12 - 16 = -4 < 0$$

Dus toenemende daling.

42 $f(x) = \frac{2x+1}{x^2+1}$ geeft

$$f'(x) = \frac{(x^2+1) \cdot 2 - (2x+1) \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2-2x}{(x^2+1)^2} = \frac{-2x^2-2x+2}{(x^2+1)^2} \text{ en}$$

$$\begin{aligned} f''(x) &= \frac{(x^2+1)^2 \cdot (-4x-2) - (-2x^2-2x+2) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4} \\ &= \frac{(x^2+1)(-4x-2) - 4x(-2x^2-2x+2)}{(x^2+1)^3} = \frac{-4x^3-2x^2-4x-2+8x^3+8x^2-8x}{(x^2+1)^3} \\ &= \frac{4x^3+6x^2-12x-2}{(x^2+1)^3} \end{aligned}$$

$$f'(-3) = \frac{-18+6+2}{10^2} < 0 \text{ en } f''(-3) = \frac{-108+54+36-2}{10^3} < 0, \text{ dus in } A \text{ toenemend dalend.}$$

$$f'(0) = \frac{0+0+2}{1^2} > 0 \text{ en } f''(0) = \frac{0+0+0-2}{1^3} < 0, \text{ dus in } B \text{ afnemend stijgend.}$$

$$f'(1) = \frac{-2-2+2}{2^2} < 0 \text{ en } f''(1) = \frac{4+6-12-2}{2^3} < 0, \text{ dus in } C \text{ toenemend dalend.}$$

Bladzijde 113

43 a $100e^{-0,01t^2} = 50$

$$e^{-0,01t^2} = \frac{1}{2}$$

$$-0,01t^2 = \ln\left(\frac{1}{2}\right)$$

$$-0,01t^2 = -\ln(2)$$

$$t^2 = 100\ln(2)$$

$$t = \sqrt{100\ln(2)} = 8,325\dots$$

Na 8,325... · 60 ≈ 500 seconden is de helft weggestroomd.

b $\frac{dV}{dt} = 100e^{-0,01t^2} \cdot -0,02t = -2te^{-0,01t^2}$

$$\frac{d}{dt} \left(\frac{dV}{dt} \right) = -2 \cdot e^{-0,01t^2} - 2t \cdot e^{-0,01t^2} \cdot -0,02t = -2e^{-0,01t^2} + 0,04t^2 e^{-0,01t^2} = (0,04t^2 - 2)e^{-0,01t^2}$$

$$t = 7 \text{ geeft } \frac{d}{dt} \left(\frac{dV}{dt} \right) \approx -0,02 < 0$$

Op $t = 7$ is V , de resterende hoeveelheid water, toenemend dalend.

Dus is $100 - V$, de hoeveelheid water die uitstroomt, toenemend stijgend.

De uitstroomsnelheid neemt dus toe.

Alternatieve uitwerking

Noem de hoeveelheid water die het vat uitstroomt H .

Op $t = 0$ is $V = 100$, dus $H = 100 - V = 100 - 100e^{-0,01t^2}$.

$$\frac{dH}{dt} = -100e^{-0,01t^2} \cdot -0,02t = 2te^{-0,01t^2}$$

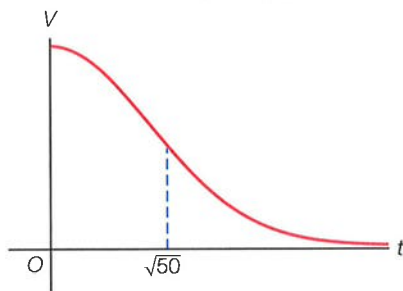
$$\frac{d}{dt}\left(\frac{dH}{dt}\right) = 2 \cdot e^{-0,01t^2} + 2t \cdot e^{-0,01t^2} \cdot -0,02t = 2e^{-0,01t^2} - 0,04t^2 e^{-0,01t^2} = (2 - 0,04t^2)e^{-0,01t^2}$$

$$t = 7 \text{ geeft } \frac{d}{dt}\left(\frac{dH}{dt}\right) \approx 0,02 > 0$$

Op $t = 7$ is de uitstroom dus toenemend stijgend.

De uitstroomsnelheid neemt dus toe.

$$\begin{aligned} \text{c } \frac{d}{dt}\left(\frac{dV}{dt}\right) = 0 & \text{ geeft } (0,04t^2 - 2)e^{-0,01t^2} = 0 \\ & 0,04t^2 - 2 = 0 \\ & 0,04t^2 = 2 \\ & t^2 = 50 \\ & t = \sqrt{50} \end{aligned}$$



De uitstroomsnelheid is maximaal na $\sqrt{50} \cdot 60 \approx 424$ seconden.

$$\text{d } \left[\frac{dV}{dt}\right]_{t=\sqrt{50}} = 8,577\dots$$

De maximale uitstroomsnelheid is dus $8,577\dots$ L/minuut.

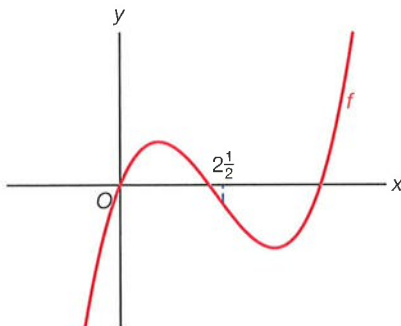
De helft van de maximale snelheid is $4,288\dots$ L/minuut.

Voer in $y_1 = -2xe^{-0,01x^2}$ en $y_2 = -4,288\dots$

Voor $x > \sqrt{50}$ geeft de optie snijpunt $x = 13,587\dots$

$(13,587\dots - \sqrt{50}) \cdot 60 \approx 391$ seconden na het in c berekende tijdstip is de uitstroomsnelheid afgenomen tot de helft van de maximale snelheid.

$$\begin{aligned} 44 \quad f(x) &= 2x^3 - 15x^2 + 24x \\ f'(x) &= 6x^2 - 30x + 24 \\ f''(x) &= 12x - 30 \\ f''(2\frac{1}{2}) &= 0 \text{ en } f'(2\frac{1}{2}) = -13\frac{1}{2} < 0 \end{aligned}$$



Dus de grafiek van f gaat bij $x = 2\frac{1}{2}$ over van toenemend dalend naar afnemend dalend.

- 45 De schets is nodig om te laten zien dat de grafiek overgaat van toenemend stijgend naar afnemend stijgend, en niet bijvoorbeeld overgaat van toenemend dalend naar afnemend dalend.

Bladzijde 114

46 a $f(x) = ax^4 - \frac{1}{6}x^3 - 3x^2 + 4x + b$
 $f'(x) = 4ax^3 - \frac{1}{2}x^2 - 6x + 4$
 $f''(x) = 12ax^2 - x - 6$
 $f''(3) = 0$ geeft $12a \cdot 3^2 - 3 - 6 = 0$

$$108a = 9$$

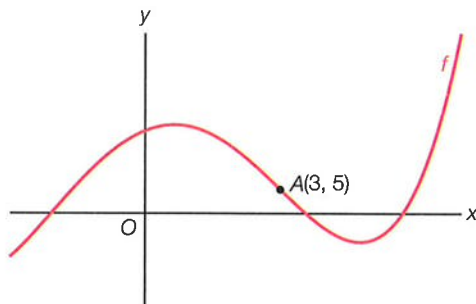
$$a = \frac{1}{12}$$

$$f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - 3x^2 + 4x + b$$

$$f(3) = 5 \text{ geeft } \frac{1}{12} \cdot 3^4 - \frac{1}{6} \cdot 3^3 - 3 \cdot 3^2 + 4 \cdot 3 + b = 5$$

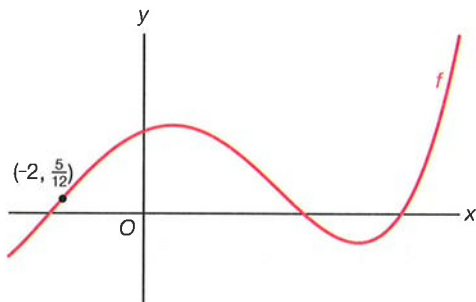
$$6\frac{3}{4} - 4\frac{1}{2} - 27 + 12 + b = 5$$

$$b = 17\frac{3}{4}$$



De grafiek van f gaat voor $a = \frac{1}{12}$ en $b = 17\frac{3}{4}$ in $A(3, 5)$ over van toenemend dalend naar afnemend dalend.

b $a = \frac{1}{12}$ geeft $f''(x) = x^2 - x - 6$
 $f''(x) = 0$ geeft $x^2 - x - 6 = 0$
 $(x+2)(x-3) = 0$
 $x = -2 \vee x = 3$



$f(-2) = \frac{5}{12}$, dus in het punt $(-2, \frac{5}{12})$ gaat de grafiek van f over van toenemend stijgend naar afnemend stijgend.

47 $f_p(x) = \frac{10}{x^2 + p}$
 $f_p'(x) = \frac{(x^2 + p) \cdot 0 - 10 \cdot 2x}{(x^2 + p)^2} = \frac{-20x}{(x^2 + p)^2}$
 $f_p''(x) = \frac{(x^2 + p)^2 \cdot -20 - (-20x) \cdot 2(x^2 + p) \cdot 2x}{(x^2 + p)^4} = \frac{(x^2 + p) \cdot -20 + 20x \cdot 2 \cdot 2x}{(x^2 + p)^3}$

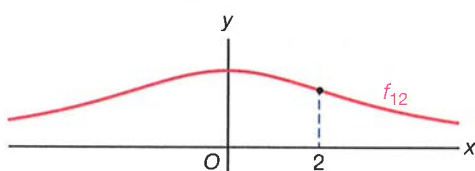
$$= \frac{-20x^2 - 20p + 80x^2}{(x^2 + p)^3} = \frac{60x^2 - 20p}{(x^2 + p)^3}$$

$$f_p''(2) = 0 \text{ geeft } \frac{60 \cdot 4 - 20p}{(4 + p)^3} = 0$$

$$240 - 20p = 0$$

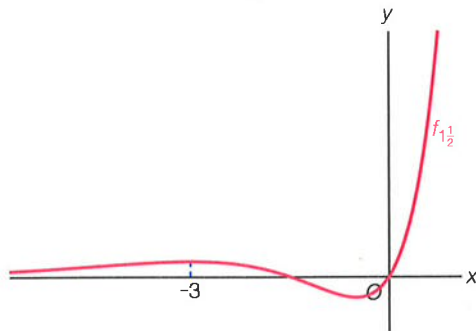
$$-20p = -240$$

$$p = 12$$



Dus voor $p = 12$ gaat de grafiek van f_p bij $x = 2$ over van toenemend dalend naar afnemend dalend.

48 a $f_a(x) = (x^2 + ax)e^x$ geeft $f'_a(x) = (2x + a) \cdot e^x + (x^2 + ax) \cdot e^x = (x^2 + (a+2)x + a)e^x$
 $f'_a(-3) = 0$ geeft $(9 - 3(a+2) + a)e^{-3} = 0$
 $9 - 3a - 6 + a = 0$
 $-2a = -3$
 $a = 1\frac{1}{2}$



Dus voor $a = 1\frac{1}{2}$ heeft f_a een maximum voor $x = -3$.

$$f_{1\frac{1}{2}}'(x) = (x^2 + 3\frac{1}{2}x + 1\frac{1}{2})e^x$$

$$f_{1\frac{1}{2}}'(x) = 0 \text{ geeft } (x^2 + 3\frac{1}{2}x + 1\frac{1}{2})e^x = 0$$

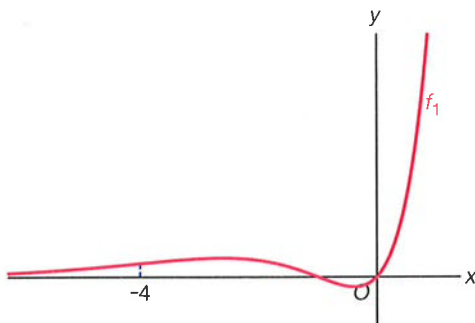
$$x^2 + 3\frac{1}{2}x + 1\frac{1}{2} = 0$$

$$(x+3)(x+\frac{1}{2}) = 0$$

$$x = -3 \vee x = -\frac{1}{2}$$

$$\text{min. is } f_{1\frac{1}{2}}(-\frac{1}{2}) = ((-\frac{1}{2})^2 + 1\frac{1}{2} \cdot -\frac{1}{2})e^{-\frac{1}{2}} = -\frac{1}{2}e^{-\frac{1}{2}} = -\frac{1}{2\sqrt{e}}$$

b $f_a'(x) = (x^2 + (a+2)x + a)e^x$ geeft
 $f_a''(x) = (2x + a + 2)e^x + (x^2 + (a+2)x + a)e^x = (x^2 + (a+4)x + 2a + 2)e^x$
 $f_a''(-4) = 0$ geeft $((-4)^2 - 4(a+4) + 2a + 2)e^{-4} = 0$
 $16 - 4a - 16 + 2a + 2 = 0$
 $-2a = -2$
 $a = 1$



Dus voor $a = 1$ gaat de grafiek van f_a bij $x = -4$ over van toenemend stijgend naar afnemend stijgend.

49 $f(-2) = -\frac{4}{e}$ geeft $-2ae^{-2b} = -\frac{4}{e}$
 $f(x) = axe^{bx}$ geeft $f'(x) = a \cdot e^{bx} + ax \cdot e^{bx} \cdot b = (abx + a)e^{bx}$
 $f'(-2) = 0$ geeft $(-2ab + a)e^{-2b} = 0$
 $-2ab + a = 0$
 $a(-2b + 1) = 0$
 $a = 0 \vee 2b = 1$
vold. niet $b = \frac{1}{2}$
 $b = \frac{1}{2}$ en $-2ae^{-2b} = -\frac{4}{e}$ geeft $-2ae^{-1} = -\frac{4}{e}$
 $-2a = -4$
 $a = 2$

$$f(x) = 2xe^{\frac{1}{2}x} \text{ en } f'(x) = (x+2)e^{\frac{1}{2}x} \text{ geeft}$$

$$f''(x) = 1 \cdot e^{\frac{1}{2}x} + (x+2) \cdot e^{\frac{1}{2}x} \cdot \frac{1}{2} = e^{\frac{1}{2}x} + (\frac{1}{2}x + 1)e^{\frac{1}{2}x} = (\frac{1}{2}x + 2)e^{\frac{1}{2}x}$$

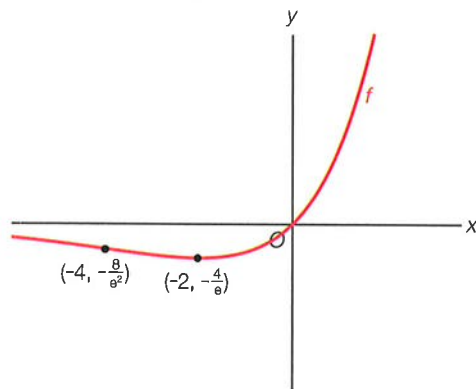
$$f'''(x) = 0 \text{ geeft } \left(\frac{1}{2}x + 2\right)e^{\frac{1}{2}x} = 0$$

$$\frac{1}{2}x + 2 = 0$$

$$\frac{1}{2}x = -2$$

$$x = -4$$

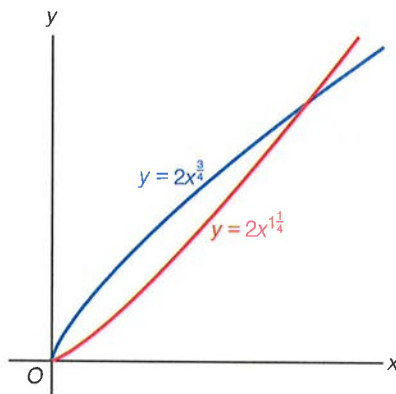
$$f(-4) = -8e^{-2} = -\frac{8}{e^2}$$



Dus grafiek van $f_{a,b}$ met top $\left(-2, -\frac{4}{e}\right)$ gaat in het punt $\left(-4, -\frac{8}{e^2}\right)$ over van toenemend dalend naar afnemend dalend.

50

a



b $y = 2x^p$ geeft $y' = 2px^{p-1}$ en $y'' = 2p(p-1)x^{p-2}$

Afnemend stijgend als $y' > 0$ en $y'' < 0$.

$$y' > 0 \text{ geeft } p > 0$$

$$y'' < 0 \text{ geeft } 0 < p < 1$$

Dus de grafiek van $y = 2x^p$ is afnemend stijgend voor $0 < p < 1$.

Bladzijde 115

51

$$y = \frac{375}{x\sqrt{x}} = \frac{375}{x^{1\frac{1}{2}}} = 375x^{-1\frac{1}{2}} \text{ geeft } y' = -562,5x^{-2\frac{1}{2}} \text{ en } y'' = 1406,25x^{-3\frac{1}{2}}$$

Omdat voor $x > 0$ geldt $x^{-2\frac{1}{2}} > 0$ en $x^{-3\frac{1}{2}} > 0$, is $y' < 0$ en $y'' > 0$.

Dus y is afnemend dalend.

52

$$\left. \begin{array}{l} y = ax\sqrt{x} \\ x = 25 \text{ en } y = 3 \end{array} \right\} \begin{array}{l} 25a \cdot \sqrt{25} = 3 \\ 25a \cdot 5 = 3 \\ 125a = 3 \\ a = 0,024 \end{array}$$

$$\text{Dus } y = 0,024x\sqrt{x}.$$

$$y = 0,024x^{1\frac{1}{2}} \text{ geeft } y' = 0,036x^{\frac{1}{2}} \text{ en } y'' = 0,018x^{-\frac{1}{2}}$$

Omdat voor $x > 0$ geldt $x^{\frac{1}{2}} > 0$ en $x^{-\frac{1}{2}} > 0$, is $y' > 0$ en $y'' > 0$.

Dus y is toenemend stijgend.

$$53 \quad a \quad \left. \begin{aligned} H &= \frac{a}{W^{0,25}} \text{ oftewel } a = H \cdot W^{0,25} \\ W &= 750 \text{ en } H = 40 \end{aligned} \right\} a = 40 \cdot 750^{0,25} \approx 209$$

$$\text{Dus } H = \frac{209}{W^{0,25}}.$$

$$b \quad H = 200 \text{ geeft } \frac{210}{W^{0,25}} = 200$$

$$W^{0,25} = \frac{210}{200}$$

$$W = \left(\frac{210}{200} \right)^4 \approx 1,2$$

Het gevraagde gewicht is 1,2 kg.

$$c \quad H = \frac{210}{W^{0,25}} = 210W^{-0,25} \text{ geeft } \frac{dH}{dW} = -52,5W^{-1,25} \text{ en } \frac{d}{dW} \left(\frac{dH}{dW} \right) = 65,625W^{-2,25}$$

Omdat voor elke $W > 0$ geldt $W^{-1,25} > 0$ en $W^{-2,25} > 0$, is $\frac{dH}{dW} < 0$ en $\frac{d}{dW} \left(\frac{dH}{dW} \right) > 0$.

Dus H is een afnemend dalende functie van W .

$$54 \quad E = aG^{-0,4} \text{ geeft } \frac{dE}{dG} = -0,4aG^{-1,4} \text{ en } \frac{d}{dG} \left(\frac{dE}{dG} \right) = 0,56aG^{-2,4}$$

Omdat voor elke $a > 0$ en $G > 0$ geldt $aG^{-1,4} > 0$ en $aG^{-2,4} > 0$, is $\frac{dE}{dG} < 0$ en $\frac{d}{dG} \left(\frac{dE}{dG} \right) > 0$.

Dus E is een afnemend dalende functie van G .

$$55 \quad a \quad \text{Neem } 3d \text{ voor } d \text{ in de formule } L = \frac{a}{d^2}.$$

$$\text{Je krijgt } L = \frac{a}{(3d)^2} = \frac{a}{9d^2} = \frac{1}{9} \cdot \frac{a}{d^2}.$$

Dus de geluidsterkte wordt 9 keer zo klein.

$$b \quad L = \frac{a}{d^2} = ad^{-2} \text{ geeft } \frac{dL}{dd} = -2ad^{-3} \text{ en } \frac{d}{dd} \left(\frac{dL}{dd} \right) = 6ad^{-4}$$

Omdat voor elke $a > 0$ en $d > 0$ geldt $ad^{-3} > 0$ en $ad^{-4} > 0$, is $\frac{dL}{dd} < 0$ en $\frac{d}{dd} \left(\frac{dL}{dd} \right) > 0$.

Dus L is een afnemend dalende functie van d .

Dus als de afstand tot een geluidsbron toeneemt, neemt de afname van de geluidsterkte af.

$$56 \quad a \quad \left. \begin{aligned} A &= aT^b \\ A &= 108 \text{ en } T = 225 \end{aligned} \right\} a \cdot 225^b = 108$$

$$a = \frac{108}{225^b}$$

$$\left. \begin{aligned} A &= aT^b \\ A &= 1427 \text{ en } T = 10753 \end{aligned} \right\} a \cdot 10753^b = 1427$$

$$a = \frac{1427}{10753^b}$$

$$\text{Voer in } y_1 = \frac{108}{225^x} \text{ en } y_2 = \frac{1427}{10753^x}.$$

De optie snijpunt geeft $x \approx 0,668$ en $y \approx 2,906$.

$b \approx 0,668$ en $a \approx 2,906$ geeft $A = 2,906T^{0,668}$

$$b \quad A = 2,91T^{\frac{2}{3}} \text{ geeft } \frac{dA}{dT} = 1,94T^{-\frac{1}{3}} \text{ en } \frac{d}{dT} \left(\frac{dA}{dT} \right) \approx -0,647T^{-\frac{4}{3}}$$

Omdat voor $T > 0$ geldt $T^{-\frac{1}{3}} > 0$ en $T^{-\frac{4}{3}} > 0$, is $\frac{dA}{dT} > 0$ en $\frac{d}{dT} \left(\frac{dA}{dT} \right) < 0$.

Dus A is een afnemend stijgende functie van T .

$$c \quad 19,3030 \text{ AE} = 2887728800 \text{ km}$$

$$A = 2887,7288 \text{ geeft } 2,91T^{\frac{2}{3}} = 2887,7288$$

$$T^{\frac{2}{3}} = \frac{2887,7288}{2,91}$$

$$T = \left(\frac{2887,7288}{2,91} \right)^{\frac{3}{2}} = 31260,4...$$

Dus de omwentelingstijd is $\frac{31260,4...}{365} \approx 85,6$ jaar.

Bladzijde 116

57 720 km/uur is $\frac{720}{3,6} = 200$ m/s.

$$\left. \begin{array}{l} W_1 = av^2 \\ v = 200 \text{ en } W_1 = 120\,000 \end{array} \right\} \begin{array}{l} a \cdot 200^2 = 120\,000 \\ a = \frac{120\,000}{200^2} = 3 \end{array}$$

Dus $W_1 = 3v^2$.

$$\left. \begin{array}{l} W_i = \frac{b}{v^2} \\ v = 200 \text{ en } W_i = 300\,000 \end{array} \right\} \begin{array}{l} \frac{b}{200^2} = 300\,000 \\ b = 300\,000 \cdot 200^2 = 1,2 \cdot 10^{10} \end{array}$$

Dus $W_i = \frac{1,2 \cdot 10^{10}}{v^2}$.

$$W = W_1 + W_i = 3v^2 + \frac{1,2 \cdot 10^{10}}{v^2} = 3v^2 + 1,2 \cdot 10^{10} \cdot v^{-2} \text{ geeft } \frac{dW}{dv} = 6v - 2,4 \cdot 10^{10} \cdot v^{-3} = 6v - \frac{2,4 \cdot 10^{10}}{v^3}$$

$$\frac{dW}{dv} = 0 \text{ geeft } 6v - \frac{2,4 \cdot 10^{10}}{v^3} = 0$$

$$6v = \frac{2,4 \cdot 10^{10}}{v^3}$$

$$6v^4 = 2,4 \cdot 10^{10}$$

$$v^4 = 4 \cdot 10^9$$

$$v = 251,48\dots$$

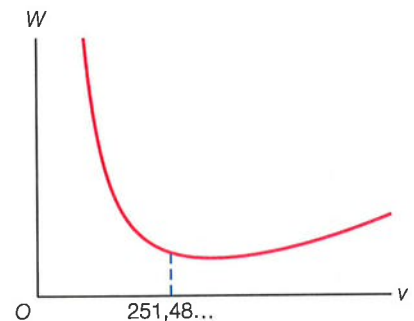
Schets van W , zie hiernaast.

Dus voor $v = 251,48\dots$ is W minimaal.

$$v = 251,48\dots \text{ geeft } W_1 = 3 \cdot 251,48\dots^2 \approx 189\,737 \text{ N}$$

$$v = 251,48\dots \text{ geeft } W_i = \frac{1,2 \cdot 10^{10}}{251,48\dots^2} \approx 189\,737 \text{ N}$$

Dus bij minimale totale weerstand geldt $W_1 = W_i$.



58 a $H = a \cdot \frac{\ln(2t+1)}{2t+1}$ geeft $\frac{dH}{dt} = a \cdot \frac{(2t+1) \cdot \frac{1}{2t+1} \cdot 2 - \ln(2t+1) \cdot 2}{(2t+1)^2} = a \cdot \frac{2 - 2\ln(2t+1)}{(2t+1)^2}$

$$\frac{dH}{dt} = 0 \text{ geeft } a \cdot \frac{2 - 2\ln(2t+1)}{(2t+1)^2} = 0$$

$$2 - 2\ln(2t+1) = 0$$

$$\ln(2t+1) = 1$$

$$2t+1 = e$$

$$2t = e - 1$$

$$t = \frac{1}{2}e - \frac{1}{2}$$

Voor $t = \frac{1}{2}e - \frac{1}{2}$ is $H = 10\,000$.

Dit geeft $a \cdot \frac{\ln(2 \cdot (\frac{1}{2}e - \frac{1}{2}) + 1)}{2 \cdot (\frac{1}{2}e - \frac{1}{2}) + 1} = 10\,000$

$$a \cdot \frac{\ln(e - 1 + 1)}{e - 1 + 1} = 10\,000$$

$$a \cdot \frac{\ln(e)}{e} = 10\,000$$

$$a \cdot \frac{1}{e} = 10\,000$$

$$a = 10\,000e$$

Dus $H = 10\,000e \cdot \frac{\ln(2t+1)}{2t+1}$.

b $a = 10\,000e$ geeft $\frac{dH}{dt} = 10\,000e \cdot \frac{2 - 2\ln(2t+1)}{(2t+1)^2}$ (zie a)

$$\left[\frac{dH}{dt} \right]_{t=2} = 10\,000e \cdot \frac{2 - 2\ln(4+1)}{(4+1)^2} \approx -1325$$

Op $t = 2$ neemt de hoeveelheid water die per uur wegstroomt af met 1325 m^3 per uur.

15.4 Integralen bij oppervlakte en inhoud

Bladzijde 118

59 $y = \frac{1}{3}(2x-3)\sqrt{2x-3} = \frac{1}{3}(2x-3)^{1\frac{1}{2}}$ geeft $\frac{dy}{dx} = \frac{1}{3} \cdot 1\frac{1}{2}(2x-3)^{\frac{1}{2}} \cdot 2 = (2x-3)^{\frac{1}{2}} = \sqrt{2x-3}$

Dus $y = \frac{1}{3}(2x-3)\sqrt{2x-3}$ is een primitieve van $y = \sqrt{2x-3}$.

Dus bewering II is juist.

Bladzijde 119

60 $f(x) = g(x)$ geeft $\sqrt{2x-3} = \frac{1}{2}x$

$$2x-3 = \frac{1}{4}x^2$$

$$-\frac{1}{4}x^2 + 2x - 3 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x = 2 \vee x = 6$$

$$\begin{aligned} O(V) &= \int_2^6 (f(x) - g(x)) dx = \int_2^6 (\sqrt{2x-3} - \frac{1}{2}x) dx = \int_2^6 ((2x-3)^{\frac{1}{2}} - \frac{1}{2}x) dx = \left[\frac{1}{2} \cdot \frac{2}{3}(2x-3)^{\frac{3}{2}} - \frac{1}{4}x^2 \right]_2^6 \\ &= \left[\frac{1}{3}(2x-3)\sqrt{2x-3} - \frac{1}{4}x^2 \right]_2^6 = \frac{1}{3} \cdot 9 \cdot \sqrt{9} - \frac{1}{4} \cdot 36 - \left(\frac{1}{3} \cdot 1 \cdot \sqrt{1} - \frac{1}{4} \cdot 4 \right) = \frac{2}{3} \end{aligned}$$

61 $f(x) = g(x)$ geeft $\frac{5}{4x-2} = -\frac{1}{2}x + 3$

$$(-\frac{1}{2}x + 3)(4x-2) = 5$$

$$-2x^2 + x + 12x - 6 = 5$$

$$-2x^2 + 13x - 11 = 0$$

$$D = 13^2 - 4 \cdot -2 \cdot -11 = 81$$

$$x = \frac{-13 \pm 9}{-4} = 1 \vee x = \frac{-13 - 9}{-4} = 5\frac{1}{2}$$

$$\begin{aligned} O(V) &= \int_1^{5\frac{1}{2}} (g(x) - f(x)) dx = \int_1^{5\frac{1}{2}} \left(-\frac{1}{2}x + 3 - \frac{5}{4x-2} \right) dx = \left[-\frac{1}{4}x^2 + 3x - 5 \cdot \frac{1}{4} \ln|4x-2| \right]_1^{5\frac{1}{2}} \\ &= -\frac{1}{4} \cdot (5\frac{1}{2})^2 + 3 \cdot 5\frac{1}{2} - 1\frac{1}{4} \ln(4 \cdot 5\frac{1}{2} - 2) - \left(-\frac{1}{4} + 3 - 1\frac{1}{4} \ln(2) \right) = -7\frac{9}{16} + 16\frac{1}{2} - 1\frac{1}{4} \ln(20) + \frac{1}{4} - 3 + 1\frac{1}{4} \ln(2) \\ &= 6\frac{3}{16} - 1\frac{1}{4} \ln(10) \end{aligned}$$

Bladzijde 120

62 $f(x) = g(x)$ geeft $\frac{x+8}{x+1} = \frac{4x-4}{x+1}$

$$x+8 = 4x-4 \wedge x+1 \neq 0$$

$$-3x = -12 \wedge x \neq -1$$

$$x = 4 \wedge x \neq -1$$

$$x = 4$$

$g(x) = 0$ geeft $4x-4 = 0$

$$4x = 4$$

$$x = 1$$

$$f(x) = \frac{x+8}{x+1} = \frac{x+1-1+8}{x+1} = 1 + \frac{7}{x+1}$$

$$g(x) = \frac{4x-4}{x+1} = \frac{4(x+1)-4-4}{x+1} = 4 - \frac{8}{x+1}$$

$$\begin{aligned} O(V) &= \int_0^1 f(x) dx + \int_1^4 (f(x) - g(x)) dx = \int_0^1 \left(1 + \frac{7}{x+1} \right) dx + \int_1^4 \left(1 + \frac{7}{x+1} - 4 + \frac{8}{x+1} \right) dx \\ &= \int_0^1 \left(1 + \frac{7}{x+1} \right) dx + \int_1^4 \left(-3 + \frac{15}{x+1} \right) dx = [x + 7 \ln|x+1|]_0^1 + [-3x + 15 \ln|x+1|]_1^4 \\ &= 1 + 7 \ln(2) - (0 + 7 \ln(1)) - 12 + 15 \ln(5) - (-3 + 15 \ln(2)) \\ &= 1 + 7 \ln(2) - 12 + 15 \ln(5) + 3 - 15 \ln(2) = 15 \ln(5) - 8 \ln(2) - 8 \end{aligned}$$

$$63 \quad f_a(x) = \frac{ax+4}{2x-3} = \frac{\frac{1}{2}a(2x-3) + 1\frac{1}{2}a+4}{2x-3} = \frac{1}{2}a + \frac{1\frac{1}{2}a+4}{2x-3}$$

$$\begin{aligned} O(V) &= \int_2^4 \left(\frac{1}{2}a + \frac{1\frac{1}{2}a+4}{2x-3} \right) dx = \left[\frac{1}{2}ax + (1\frac{1}{2}a+4) \cdot \frac{1}{2} \ln|2x-3| \right]_2^4 \\ &= 2a + (1\frac{1}{2}a+4) \cdot \frac{1}{2} \ln(5) - (a + (1\frac{1}{2}a+4) \cdot \frac{1}{2} \ln(1)) = 2a + (1\frac{1}{2}a+4) \cdot \frac{1}{2} \ln(5) - a - 0 \\ &= a + \frac{3}{4}a \ln(5) + 2 \ln(5) \end{aligned}$$

$$O(V) = 10 \text{ geeft } a + \frac{3}{4}a \ln(5) + 2 \ln(5) = 10$$

$$a(1 + \frac{3}{4} \ln(5)) = 10 - 2 \ln(5)$$

$$a = \frac{10 - 2 \ln(5)}{1 + \frac{3}{4} \ln(5)} \approx 3,07$$

$$64 \quad f(x) = g(x) \text{ geeft } \sin^2(x) = \frac{1}{2} \sin(2x) \\ \sin^2(x) = \sin(x) \cos(x) \\ \sin(x) = 0 \vee \sin(x) = \cos(x)$$

$$\sin(x) = \cos(x) \text{ geeft } \tan(x) = 1$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$$\text{Dus } x_A = \frac{1}{4}\pi.$$

$$\cos(2x) = 1 - 2 \sin^2(x), \text{ dus } 2 \sin^2(x) = 1 - \cos(2x) \text{ oftewel } \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x).$$

$$\text{Dus } f(x) = \frac{1}{2} - \frac{1}{2} \cos(2x).$$

$$\begin{aligned} O(V) &= \int_0^{\frac{1}{4}\pi} (g(x) - f(x)) dx = \int_0^{\frac{1}{4}\pi} \left(\frac{1}{2} \sin(2x) - \frac{1}{2} + \frac{1}{2} \cos(2x) \right) dx = \left[-\frac{1}{4} \cos(2x) - \frac{1}{2}x + \frac{1}{4} \sin(2x) \right]_0^{\frac{1}{4}\pi} \\ &= -\frac{1}{4} \cdot 0 - \frac{1}{8}\pi + \frac{1}{4} \cdot 1 - \left(-\frac{1}{4} \cdot 1 - \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot 0 \right) = \frac{1}{2} - \frac{1}{8}\pi \end{aligned}$$

$$\begin{aligned} O(W) &= \int_{\frac{1}{4}\pi}^{\pi} (f(x) - g(x)) dx = \int_{\frac{1}{4}\pi}^{\pi} \left(\frac{1}{2} - \frac{1}{2} \cos(2x) - \frac{1}{2} \sin(2x) \right) dx = \left[\frac{1}{2}x - \frac{1}{4} \sin(2x) + \frac{1}{4} \cos(2x) \right]_{\frac{1}{4}\pi}^{\pi} \\ &= \frac{1}{2}\pi - \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 1 - \left(\frac{1}{8}\pi - \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0 \right) = \frac{1}{2} + \frac{3}{8}\pi \end{aligned}$$

$$O(V) + O(W) = \frac{1}{2} - \frac{1}{8}\pi + \frac{1}{2} + \frac{3}{8}\pi = 1 + \frac{1}{4}\pi$$

Bladzijde 121

$$65 \quad \mathbf{a} \quad O(V+W) = \int_0^p \sqrt{x} dx = \int_0^p x^{\frac{1}{2}} dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^p = \left[\frac{2}{3} x \sqrt{x} \right]_0^p = \frac{2}{3} p \sqrt{p} - 0 = \frac{2}{3} p \sqrt{p}$$

$$\mathbf{b} \quad O(W) = \frac{1}{2} \cdot p \cdot f(p) = \frac{1}{2} p \sqrt{p}$$

$$\mathbf{c} \quad O(V) = O(V+W) - O(W) = \frac{2}{3} p \sqrt{p} - \frac{1}{2} p \sqrt{p} = \frac{1}{6} p \sqrt{p}$$

$$\mathbf{d} \quad O(V) : O(W) = \frac{1}{6} p \sqrt{p} : \frac{1}{2} p \sqrt{p} = \frac{1}{6} : \frac{1}{2} = 1 : 3$$

Bladzijde 122

$$66 \quad O(V) = O(W)$$

$$1 + 2 \ln(p) = p^2 - (1 + 2 \ln(p))$$

$$1 + 2 \ln(p) = p^2 - 1 - 2 \ln(p)$$

$$2 + 4 \ln(p) = p^2$$

$$\text{Voer in } y_1 = 2 + 4 \ln(x) \text{ en } y_2 = x^2.$$

$$\text{De optie snijpunt geeft } x \approx 0,68 \text{ en } x \approx 2,31.$$

$$x \approx 0,68 \text{ voldoet niet, dus } p \approx 2,31.$$

$$67 \quad O(V+W) = \int_{-3}^3 f(x) dx = \int_{-3}^3 (9-x^2) dx = \left[9x - \frac{1}{3}x^3 \right]_{-3}^3 = 27 - 9 - (-27 + 9) = 36$$

$$g_a(x) = 0 \text{ geeft } a - x^2 = 0$$

$$x^2 = a$$

$$x = \sqrt{a} \vee x = -\sqrt{a}$$

$$O(W) = \int_{-\sqrt{a}}^{\sqrt{a}} g_a(x) dx = \int_{-\sqrt{a}}^{\sqrt{a}} (a - x^2) dx = \left[ax - \frac{1}{3}x^3 \right]_{-\sqrt{a}}^{\sqrt{a}} = a\sqrt{a} - \frac{1}{3}a\sqrt{a} - (-a\sqrt{a} + \frac{1}{3}a\sqrt{a}) = 1\frac{1}{3}a\sqrt{a}$$

$$O(V) = O(W) \text{ geeft } O(W) = \frac{1}{2}O(V+W)$$

$$1\frac{1}{3}a\sqrt{a} = \frac{1}{2} \cdot 36$$

$$a\sqrt{a} = 13\frac{1}{2}$$

$$a^3 = 182\frac{1}{4}$$

$$a = \sqrt[3]{182\frac{1}{4}} \approx 5,67$$

$$68 \quad a \quad f_{3\frac{1}{3}}(x) = -\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x \text{ geeft } f_{3\frac{1}{3}}'(x) = -x^2 + 2x + 3\frac{1}{3}$$

$$f_{3\frac{1}{3}}'(0) = 3\frac{1}{3}, \text{ dus } k: y = 3\frac{1}{3}x.$$

$$k \text{ snijden met de grafiek van } f_{3\frac{1}{3}} \text{ geeft } -\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x = 3\frac{1}{3}x$$

$$-\frac{1}{3}x^3 + x^2 = 0$$

$$-\frac{1}{3}x^2(x-3) = 0$$

$$x = 0 \vee x = 3$$

$$f_{3\frac{1}{3}}(x) = 0 \text{ geeft } -\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x = 0$$

$$-\frac{1}{3}x(x^2 - 3x - 10) = 0$$

$$-\frac{1}{3}x(x+2)(x-5) = 0$$

$$x = 0 \vee x = -2 \vee x = 5$$

$$O(W) = \int_0^3 3\frac{1}{3}x dx + \int_3^5 (-\frac{1}{3}x^3 + x^2 + 3\frac{1}{3}x) dx = \left[\frac{5}{3}x^2 \right]_0^3 + \left[-\frac{1}{12}x^4 + \frac{1}{3}x^3 + \frac{5}{3}x^2 \right]_3^5$$

$$= 15 - 0 - \frac{625}{12} + \frac{125}{3} + \frac{125}{3} - (-\frac{27}{4} + 9 + 15) = 29$$

$$b \quad f_a(x) = -\frac{1}{3}x^3 + x^2 + ax \text{ geeft } f_a'(x) = -x^2 + 2x + a$$

$$f_a'(0) = a, \text{ dus } k: y = ax.$$

$$k \text{ snijden met de grafiek van } f_a \text{ geeft } -\frac{1}{3}x^3 + x^2 + ax = ax$$

$$-\frac{1}{3}x^3 + x^2 = 0$$

$$-\frac{1}{3}x^2(x-3) = 0$$

$$x = 0 \vee x = 3$$

$$O(V) = \int_0^3 (-\frac{1}{3}x^3 + x^2 + ax - ax) dx = \int_0^3 (-\frac{1}{3}x^3 + x^2) dx \text{ en dit is onafhankelijk van } a.$$

$$69 \quad f_a(x) = \frac{1}{4}x^2(x-a)^2 = \frac{1}{4}x^2(x^2 - 2ax + a^2) = \frac{1}{4}x^4 - \frac{1}{2}ax^3 + \frac{1}{4}a^2x^2 \text{ geeft } f_a'(x) = x^3 - 1\frac{1}{2}ax^2 + \frac{1}{2}a^2x$$

$$f_a'(x) = 0 \text{ geeft } x^3 - 1\frac{1}{2}ax^2 + \frac{1}{2}a^2x = 0$$

$$x(x^2 - 1\frac{1}{2}ax + \frac{1}{2}a^2) = 0$$

$$x(x - \frac{1}{2}a)(x - a) = 0$$

$$x = 0 \vee x = \frac{1}{2}a \vee x = a$$

$$f_a(\frac{1}{2}a) = \frac{1}{16}a^2(\frac{1}{2}a - a)^2 = \frac{1}{64}a^4, \text{ dus } T(\frac{1}{2}a, \frac{1}{64}a^4).$$

$$O(OABC) = a \cdot \frac{1}{64}a^4 = \frac{1}{64}a^5$$

$$O(V) = \int_0^a (\frac{1}{4}x^4 - \frac{1}{2}ax^3 + \frac{1}{4}a^2x^2) dx = \left[\frac{1}{20}x^5 - \frac{1}{8}ax^4 + \frac{1}{12}a^2x^3 \right]_0^a = \frac{1}{20}a^5 - \frac{1}{8}a^5 + \frac{1}{12}a^5 - 0 = \frac{1}{120}a^5$$

$$\text{De gevraagde verhouding is } \frac{1}{120}a^5 : \frac{1}{64}a^5 = 64 : 120 = 8 : 15.$$

70 $f_a(x) = 0$ geeft $x = a$

$$f_a(x) = 2\sqrt{x} \text{ geeft } \sqrt{a-x} = 2\sqrt{x}$$

$$a-x = 4x$$

$$-5x = -a$$

$$x = \frac{1}{5}a$$

$$f_a(x) = \frac{1}{2}\sqrt{x} \text{ geeft } \sqrt{a-x} = \frac{1}{2}\sqrt{x}$$

$$a-x = \frac{1}{4}x$$

$$-\frac{5}{4}x = -a$$

$$x = \frac{4}{5}a$$

$$O(V+W) = \int_0^{\frac{1}{5}a} 2\sqrt{x} dx + \int_{\frac{1}{5}a}^a \sqrt{a-x} dx = \int_0^{\frac{1}{5}a} 2x^{\frac{1}{2}} dx + \int_{\frac{1}{5}a}^a (a-x)^{\frac{1}{2}} dx = \left[\frac{4}{3}x^{\frac{3}{2}} \right]_0^{\frac{1}{5}a} + \left[-\frac{2}{3}(a-x)^{\frac{3}{2}} \right]_{\frac{1}{5}a}^a$$

$$= \left[\frac{4}{3}x\sqrt{x} \right]_0^{\frac{1}{5}a} + \left[-\frac{2}{3}(a-x)\sqrt{a-x} \right]_{\frac{1}{5}a}^a = \frac{4}{3} \cdot \frac{1}{5}a\sqrt{\frac{1}{5}a} - 0 + 0 - \left(-\frac{2}{3} \cdot \frac{4}{5}a\sqrt{\frac{4}{5}a} \right) = \frac{4}{15}a\sqrt{\frac{1}{5}a} + \frac{16}{15}a\sqrt{\frac{1}{5}a} = \frac{4}{3}a\sqrt{\frac{1}{5}a}$$

$$O(W) = \int_0^{\frac{4}{5}a} \frac{1}{2}\sqrt{x} dx + \int_{\frac{4}{5}a}^a \sqrt{a-x} dx = \int_0^{\frac{4}{5}a} \frac{1}{2}x^{\frac{1}{2}} dx + \int_{\frac{4}{5}a}^a (a-x)^{\frac{1}{2}} dx = \left[\frac{1}{3}x^{\frac{3}{2}} \right]_0^{\frac{4}{5}a} + \left[-\frac{2}{3}(a-x)^{\frac{3}{2}} \right]_{\frac{4}{5}a}^a$$

$$= \left[\frac{1}{3}x\sqrt{x} \right]_0^{\frac{4}{5}a} + \left[-\frac{2}{3}(a-x)\sqrt{a-x} \right]_{\frac{4}{5}a}^a = \frac{1}{3} \cdot \frac{4}{5}a\sqrt{\frac{4}{5}a} - 0 + 0 - \left(-\frac{2}{3} \cdot \frac{1}{5}a\sqrt{\frac{1}{5}a} \right) = \frac{8}{15}a\sqrt{\frac{1}{5}a} + \frac{2}{15}a\sqrt{\frac{1}{5}a} = \frac{2}{3}a\sqrt{\frac{1}{5}a}$$

$$O(W) = \frac{1}{2}O(V+W), \text{ dus } O(V) = O(W).$$

Bladzijde 123**71 a** $f(x) = 2\sqrt{x}$ geeft $f'(x) = 2 \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$$k: y = ax + b \text{ met } a = f'(p^2) = \frac{1}{\sqrt{p^2}} = \frac{1}{p}$$

$$\left. \begin{array}{l} y = \frac{1}{p} \cdot x + b \\ \text{door } P(p^2, 2p) \end{array} \right\} \begin{array}{l} \frac{1}{p} \cdot p^2 + b = 2p \\ p + b = 2p \\ b = p \end{array}$$

$$\text{Dus } k: y = \frac{1}{p} \cdot x + p.$$

b $rc_l = \frac{y_p}{x_p} = \frac{2p}{p^2} = \frac{2}{p}$ geeft $l: y = \frac{2}{p}x$

$$O(V) = \int_0^{p^2} \left(2\sqrt{x} - \frac{2}{p} \cdot x \right) dx = \int_0^{p^2} \left(2x^{\frac{1}{2}} - \frac{2}{p} \cdot x \right) dx = \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{p} \cdot x^2 \right]_0^{p^2} = \left[\frac{4}{3}x\sqrt{x} - \frac{1}{p} \cdot x^2 \right]_0^{p^2} = \frac{4}{3}p^3 - p^3 = \frac{1}{3}p^3$$

$$O(W) = \int_0^{p^2} \left(\frac{1}{p} \cdot x + p - 2\sqrt{x} \right) dx = \left[\frac{1}{2p} \cdot x^2 + px - \frac{4}{3}x\sqrt{x} \right]_0^{p^2} = \frac{1}{2}p^3 + p^3 - \frac{4}{3}p^3 = \frac{1}{6}p^3$$

$$O(W) : O(V) = \frac{1}{6}p^3 : \frac{1}{3}p^3 = 1 : 2$$

72 $D_f = \langle 0, \rightarrow \rangle$, dus $f(x) = \ln^2(x) + \ln(x^2) = \ln^2(x) + 2 \ln(x)$.

$$\int_q^{2q} g(x) dx = 0$$

$$\int_q^{2q} f'(x) dx = 0$$

$$[f(x)]_q^{2q} = 0$$

$$f(2q) - f(q) = 0$$

$$\ln^2(2q) + 2 \ln(2q) - (\ln^2(q) + 2 \ln(q)) = 0$$

$$\ln^2(2q) - \ln^2(q) + 2 \ln(2q) - 2 \ln(q) = 0$$

$$(\ln(2q) - \ln(q))(\ln(2q) + \ln(q)) + 2(\ln(2q) - \ln(q)) = 0$$

$$\ln(2) \cdot \ln(2q^2) + 2 \ln(2) = 0$$

$$\ln(2q^2) + 2 = 0$$

$$\ln(2q^2) = -2$$

$$2q^2 = e^{-2}$$

$$q^2 = \frac{1}{2e^2}$$

$$q = \sqrt{\frac{1}{2e^2}} \vee q = -\sqrt{\frac{1}{2e^2}}$$

$$q = \frac{1}{\sqrt{2} \cdot e} \quad \text{vold. niet}$$

$$\text{Dus } q = \frac{1}{\sqrt{2} \cdot e}.$$

73 a $I(L) = \pi \int_0^p (\sqrt{x})^2 dx = \pi \int_0^p x dx = \pi \left[\frac{1}{2} x^2 \right]_0^p = \frac{1}{2} \pi p^2$

b $y = \sqrt{x}$ geeft $y^2 = x$ oftewel $x = y^2$

$$I(M) = \pi \int_0^{\sqrt{p}} x^2 dy = \pi \int_0^{\sqrt{p}} y^4 dy = \pi \left[\frac{1}{5} y^5 \right]_0^{\sqrt{p}} = \frac{1}{5} \pi p^2 \cdot \sqrt{p}$$

c $I(L) = I(M)$ geeft $\frac{1}{2} \pi p^2 = \frac{1}{5} \pi p^2 \cdot \sqrt{p}$
 $\frac{1}{2} = \frac{1}{5} \sqrt{p}$
 $\sqrt{p} = \frac{5}{2}$
 $p = 6\frac{1}{4}$

Bladzijde 125

74 a $I(M) = 1\frac{2}{3} I(L)$ geeft $\left(21 - \frac{21}{a^2} \right) \pi = \frac{7}{5} \cdot 15\frac{3}{4} \pi$

$$21 - \frac{21}{a^2} = 22\frac{1}{20}$$

$$\frac{21}{a^2} = -\frac{21}{20}$$

$$a^2 = -20$$

$a^2 = -20$ heeft geen oplossingen, dus $I(M) = 1\frac{2}{3} I(L)$ lukt niet.

b $I(M) = 1\frac{3}{10} I(L)$ geeft $\left(21 - \frac{21}{a^2} \right) \pi = \frac{13}{10} \cdot 15\frac{3}{4} \pi$

$$21 - \frac{21}{a^2} = 20\frac{19}{40}$$

$$\frac{21}{a^2} = \frac{21}{40}$$

$$a^2 = 40$$

$a^2 = 40$ heeft twee oplossingen, dus $I(M) = 1\frac{3}{10} I(L)$ lukt wel.

75 a $f(x) = g_a(x)$ geeft $x^2 = a\sqrt{x}$

$$x^{1\frac{1}{2}} = a$$

$$x = a^{\frac{2}{3}}$$

$$x = \sqrt[3]{a^2}$$

$$O(V) = \int_0^{\sqrt[3]{a^2}} (a\sqrt{x} - x^2) dx = \int_0^{\sqrt[3]{a^2}} (ax^{\frac{1}{2}} - x^2) dx = \left[\frac{2}{3}ax^{\frac{3}{2}} - \frac{1}{3}x^3 \right]_0^{\sqrt[3]{a^2}} = \frac{2}{3}a^2 - \frac{1}{3}a^2 = \frac{1}{3}a^2$$

$$O(V) = 10 \text{ geeft } \frac{1}{3}a^2 = 10$$

$$a^2 = 30$$

$$a = \sqrt{30} \vee a = -\sqrt{30}$$

vold. niet

$$\text{Dus } a = \sqrt{30}.$$

b $I(L) = \pi \int_0^{\sqrt[3]{a^2}} (a^2x - x^4) dx = \pi \left[\frac{1}{2}a^2x^2 - \frac{1}{5}x^5 \right]_0^{\sqrt[3]{a^2}} = \pi \left(\frac{1}{2}a^2 \cdot a^{1\frac{1}{3}} - \frac{1}{5}a^{3\frac{1}{3}} \right) = \frac{3}{10}\pi a^{3\frac{1}{3}}$

$$I(L) = 30\pi \text{ geeft } \frac{3}{10}\pi a^{3\frac{1}{3}} = 30\pi$$

$$a^{3\frac{1}{3}} = 100$$

$$a = 100^{\frac{3}{10}}$$

$$a = \sqrt[10]{1\,000\,000}$$

Bladzijde 126

76 a Is de top van p_2 het punt $T(t, t^2)$, dan is $(2t, 0)$ het andere snijpunt van p_2 met de x -as.
Dus $p_2: y = ax(x - 2t)$.

$$\left. \begin{array}{l} y = ax(x - 2t) \\ \text{door } (t, t^2) \end{array} \right\} \begin{array}{l} at(t - 2t) = t^2 \\ at \cdot -t = t^2 \\ a = -1 \end{array}$$

$$\text{Dus } p_2: y = -x(x - 2t) \text{ oftewel } p_2: y = -x^2 + 2tx.$$

b $O(V) = \int_0^t (-x^2 + 2tx - x^2) dx = \int_0^t (-2x^2 + 2tx) dx = \left[-\frac{2}{3}x^3 + tx^2 \right]_0^t = -\frac{2}{3}t^3 + t^3 = \frac{1}{3}t^3$

$$O(V) = 10 \text{ geeft } \frac{1}{3}t^3 = 10$$

$$t^3 = 30$$

$$t = \sqrt[3]{30}$$

c $I(L) = \pi \int_0^t ((-x^2 + 2tx)^2 - (x^2)^2) dx = \pi \int_0^t (x^4 - 4tx^3 + 4t^2x^2 - x^4) dx = \pi \int_0^t (-4tx^3 + 4t^2x^2) dx$

$$= \pi \left[-tx^4 + \frac{4}{3}t^2x^3 \right]_0^t = \pi(-t^5 + \frac{4}{3}t^5) = \frac{1}{3}\pi t^5$$

$$I(L) = 100\pi \text{ geeft } \frac{1}{3}\pi t^5 = 100\pi$$

$$t^5 = 300$$

$$t = \sqrt[5]{300}$$

77 a $e^x = a$ geeft $x = \ln(a)$, dus $k: x = \ln(a)$.

$$O(V) = \int_0^{\ln(a)} e^x dx = [e^x]_0^{\ln(a)} = e^{\ln(a)} - e^0 = a - 1$$

$$O(W) = a \cdot \ln(a) - O(V) = a \ln(a) - (a - 1) = a \ln(a) - a + 1$$

$$O(V) = O(W) \text{ geeft } a - 1 = a \ln(a) - a + 1$$

$$2a - 2 = a \ln(a)$$

$$\text{Voer in } y_1 = 2x - 2 \text{ en } y_2 = x \ln(x).$$

$$\text{De optie snijpunt geeft } x = 1 \text{ en } x \approx 4,92.$$

$$\text{Dus } a \approx 4,92.$$

$$\text{b } I(L) = \pi \int_0^{\ln(a)} (e^x)^2 dx = \pi \int_0^{\ln(a)} e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_0^{\ln(a)} = \pi \left(\frac{1}{2} e^{2 \ln(a)} - \frac{1}{2} e^0 \right) = \pi \left(\frac{1}{2} (e^{\ln(a)})^2 - \frac{1}{2} \right) = \frac{1}{2} \pi a^2 - \frac{1}{2} \pi$$

$$I(M) = \pi \cdot a^2 \cdot \ln(a) - I(L) = \pi a^2 \ln(a) - \left(\frac{1}{2} \pi a^2 - \frac{1}{2} \pi \right) = \pi a^2 \ln(a) - \frac{1}{2} \pi a^2 + \frac{1}{2} \pi$$

$$I(L) = I(M) \text{ geeft } \frac{1}{2} \pi a^2 - \frac{1}{2} \pi = \pi a^2 \ln(a) - \frac{1}{2} \pi a^2 + \frac{1}{2} \pi$$

$$\pi a^2 - \pi = \pi a^2 \ln(a)$$

$$a^2 - 1 = a^2 \ln(a)$$

Voer in $y_1 = x^2 - 1$ en $y_2 = x^2 \ln(x)$.

De optie snijpunt geeft $x = 1$ en $x \approx 2,22$.

Dus $a \approx 2,22$.

$$\text{c } G(x) = x \ln^2(x) - 2x \ln(x) + 2x \text{ geeft}$$

$$G'(x) = 1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} - \left(2 \cdot \ln(x) + 2x \cdot \frac{1}{x} \right) + 2 = \ln^2(x) + 2 \ln(x) - 2 \ln(x) - 2 + 2 = \ln^2(x)$$

$G'(x) = g(x)$, dus G is een primitieve van g .

$$\text{d } y = e^x \text{ geeft } x = \ln(y)$$

$$I(N) = \pi \int_1^a x^2 dy = \pi \int_1^a \ln^2(y) dy = \pi [y \ln^2(y) - 2y \ln(y) + 2y]_1^a$$

$$= \pi (a \ln^2(a) - 2a \ln(a) + 2a - (\ln^2(1) - 2 \ln(1) + 2)) = \pi (a \ln^2(a) - 2a \ln(a) + 2a - 2)$$

$$I(L) = 6 \cdot I(N) \text{ geeft } \frac{1}{2} \pi a^2 - \frac{1}{2} \pi = 6\pi (a \ln^2(a) - 2a \ln(a) + 2a - 2)$$

$$a^2 - 1 = 12(a \ln^2(a) - 2a \ln(a) + 2a - 2)$$

Voer in $y_1 = x^2 - 1$ en $y_2 = 12(x \ln^2(x) - 2x \ln(x) + 2x - 2)$.

De optie snijpunt geeft $x \approx 0,51$, $x = 1$ en $x \approx 2,28$.

Dus $a \approx 2,28$.

Bladzijde 127

$$\text{78 a } y = \frac{1}{2} x^2 - 2 \text{ geeft } \frac{1}{2} x^2 = y + 2, \text{ dus } x^2 = 2y + 4.$$

$$\text{De inhoud van de afgeknotte paraboloid is } I = \pi \int_0^p x^2 dy = \pi \int_0^p (2y + 4) dy = \pi [y^2 + 4y]_0^p = \pi(p^2 + 4p).$$

$$f(x) = \frac{1}{2} p \text{ geeft } \frac{1}{2} x^2 - 2 = \frac{1}{2} p$$

$$\frac{1}{2} x^2 = \frac{1}{2} p + 2$$

$$x^2 = p + 4$$

Dus $A = \pi(p + 4)$.

Samen met hoogte p geeft dit $p \cdot A = p \cdot \pi(p + 4) = \pi(p^2 + 4p)$, en dit is gelijk aan I .

Dus voor de inhoud I van de afgeknotte paraboloid geldt $I = p \cdot A$.

$$\text{b } PQ \text{ is een middellijn van de cirkel, dus de straal van de cirkel en dus van de bol is } \sqrt{p + 4} \text{ (zie a).}$$

$$\text{Dus } I(\text{bol}) = \frac{4}{3} \pi(p + 4) \sqrt{p + 4}.$$

$$I(\text{bol}) = I(\text{afgeknotte paraboloid}) \text{ geeft } \frac{4}{3} \pi(p + 4) \sqrt{p + 4} = \pi(p^2 + 4p)$$

$$\frac{4}{3} (p + 4) \sqrt{p + 4} = p(p + 4)$$

$$p + 4 = 0 \vee \frac{4}{3} \sqrt{p + 4} = p$$

$$p = -4 \vee 4 \sqrt{p + 4} = 3p$$

$$\text{vold. niet } 16(p + 4) = 9p^2$$

$$16p + 64 = 9p^2$$

$$9p^2 - 16p - 64 = 0$$

$$D = (-16)^2 - 4 \cdot 9 \cdot -64 = 2560$$

$$p = \frac{16 + \sqrt{2560}}{18} \vee p = \frac{16 - \sqrt{2560}}{18}$$

vold. niet

$$p = \frac{16 + \sqrt{2560}}{18} \text{ geeft straal bol} = \sqrt{\frac{16 + \sqrt{2560}}{18} + 4} \approx 2,77$$

- 79** Stel de straal van de bol is R .
 Dan is de inhoud van de bol $\frac{4}{3}\pi R^3$.
 Uit de bol zijn een cilinder en twee 'kapjes' geboord.
 Zie de figuur hiernaast met een dwarsdoorsnede van de situatie in een assenstelsel.

$$\angle A = 90^\circ, \text{ dus } OA = \sqrt{R^2 - 9}.$$

$$\text{De inhoud van de cilinder is } \pi \cdot (R^2 - 9) \cdot 6 = 6\pi R^2 - 54\pi.$$

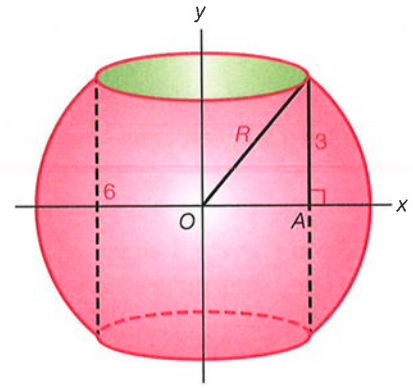
De dwarsdoorsnede van de bol is de cirkel met vergelijking $x^2 + y^2 = R^2$. Hieruit volgt $x^2 = R^2 - y^2$.

De inhoud van een kapje is

$$\begin{aligned} \pi \int_3^R x^2 dy &= \pi \int_3^R (R^2 - y^2) dy = \pi \left[R^2 y - \frac{1}{3} y^3 \right]_3^R = \pi \left(R^3 - \frac{1}{3} R^3 - (3R^2 - 9) \right) \\ &= \frac{2}{3} \pi R^3 - 3\pi R^2 + 9\pi. \end{aligned}$$

$$\begin{aligned} I(\text{bol}) - I(\text{cilinder}) - 2 \cdot I(\text{kapje}) &= \frac{4}{3} \pi R^3 - (6\pi R^2 - 54\pi) - 2 \cdot \left(\frac{2}{3} \pi R^3 - 3\pi R^2 + 9\pi \right) \\ &= \frac{4}{3} \pi R^3 - 6\pi R^2 + 54\pi - \frac{4}{3} \pi R^3 + 6\pi R^2 - 18\pi \\ &= 36\pi \end{aligned}$$

De inhoud van het deel dat er overblijft, is dus 36π .



Diagnostische toets

Bladzijde 130

- 1** Stel $x_C = a$, dan is $x_B = 2a$.

$$f(2a) = g(a) \text{ geeft } \ln\left(\frac{1}{2} \cdot 2a\right) = \ln\left(\frac{2}{3-a}\right)$$

$$\ln(a) = \ln\left(\frac{2}{3-a}\right)$$

$$a = \frac{2}{3-a}$$

$$3a - a^2 = 2$$

$$a^2 - 3a + 2 = 0$$

$$(a-1)(a-2) = 0$$

$$a = 1 \vee a = 2$$

$$p = f(2a) = f(2) = \ln(1) = 0 \text{ vold. niet}$$

$$p = f(2a) = f(4) = \ln(2)$$

- 2** $f_p(x) = \frac{10x-15}{x^2+4} + p$ geeft $f_p'(x) = \frac{(x^2+4) \cdot 10 - (10x-15) \cdot 2x}{(x^2+4)^2} = \frac{10x^2+40-20x^2+30x}{(x^2+4)^2} = \frac{-10x^2+30x+40}{(x^2+4)^2}$

$$f_p'(x) = 0 \text{ geeft } -10x^2 + 30x + 40 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x+1)(x-4) = 0$$

$$x = -1 \vee x = 4$$

$$y_A = f_p(-1) = \frac{-10-15}{1+4} + p = -5 + p \text{ en } OA^2 = (-1)^2 + (-5+p)^2 = 1 + 25 - 10p + p^2 = p^2 - 10p + 26.$$

$$y_B = f_p(4) = \frac{40-15}{16+4} + p = 1\frac{1}{4} + p \text{ en } OB^2 = 4^2 + (1\frac{1}{4}+p)^2 = 16 + 1\frac{9}{16} + 2\frac{1}{2}p + p^2 = p^2 + 2\frac{1}{2}p + 17\frac{9}{16}.$$

$$OA = OB \text{ oftewel } OA^2 = OB^2 \text{ geeft } p^2 - 10p + 26 = p^2 + 2\frac{1}{2}p + 17\frac{9}{16}$$

$$-12\frac{1}{2}p = -8\frac{7}{16}$$

$$p = \frac{27}{40}$$

3 $L = g(p) - f(p) = 3\frac{1}{2} - p - (2 - \sqrt{2p}) = 1\frac{1}{2} - p + \sqrt{2p}$ geeft

$$\frac{dL}{dp} = -1 + \frac{1}{2\sqrt{2p}} \cdot 2 = \frac{1}{\sqrt{2p}} - 1$$

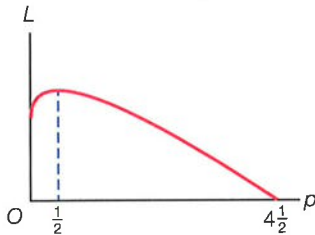
$$\frac{dL}{dp} = 0 \text{ geeft } \frac{1}{\sqrt{2p}} - 1 = 0$$

$$\frac{1}{\sqrt{2p}} = 1$$

$$\sqrt{2p} = 1$$

$$2p = 1$$

$$p = \frac{1}{2}$$



Dus voor $p = \frac{1}{2}$ is L maximaal.

4 a $A = O(\triangle OPQ) = \frac{1}{2} \cdot p \cdot (2\sqrt{p} - p) = p\sqrt{p} - \frac{1}{2}p^2 = p^{1\frac{1}{2}} - \frac{1}{2}p^2$ geeft $\frac{dA}{dp} = 1\frac{1}{2}p^{\frac{1}{2}} - p = 1\frac{1}{2}\sqrt{p} - p$

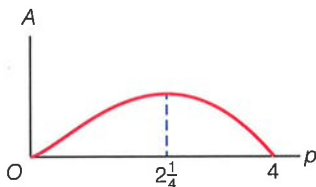
$$\frac{dA}{dp} = 0 \text{ geeft } 1\frac{1}{2}\sqrt{p} - p = 0$$

$$p = 1\frac{1}{2}\sqrt{p}$$

$$p^2 = 2\frac{1}{4}p$$

$$p = 0 \vee p = 2\frac{1}{4}$$

vold. niet



Dus voor $p = 2\frac{1}{4}$ is de oppervlakte van driehoek OPQ maximaal.

$$A_{\max} = 2\frac{1}{4}\sqrt{2\frac{1}{4}} - \frac{1}{2} \cdot (2\frac{1}{4})^2 = 2\frac{1}{4} \cdot 1\frac{1}{2} - \frac{1}{2} \cdot 5\frac{1}{16} = 3\frac{3}{8} - 2\frac{17}{32} = \frac{27}{32}$$

b $L = BP = \sqrt{(x_P - x_B)^2 + (y_P - y_B)^2} = \sqrt{(p-3)^2 + (2\sqrt{p} - p - 0)^2} = \sqrt{(p-3)^2 + (2\sqrt{p} - p)^2}$
 $= \sqrt{p^2 - 6p + 9 + 4p - 4p\sqrt{p} + p^2} = \sqrt{2p^2 - 4p\sqrt{p} - 2p + 9}$

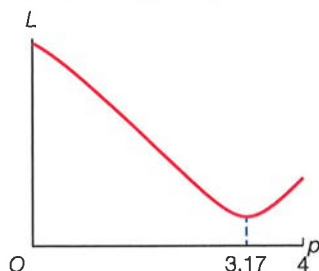
c $L = \sqrt{2p^2 - 4p\sqrt{p} - 2p + 9} = (2p^2 - 4p^{1\frac{1}{2}} - 2p + 9)^{\frac{1}{2}}$ geeft

$$\frac{dL}{dp} = \frac{1}{2}(2p^2 - 4p^{1\frac{1}{2}} - 2p + 9)^{-\frac{1}{2}} \cdot (4p - 6p^{\frac{1}{2}} - 2) = \frac{4p - 6\sqrt{p} - 2}{2\sqrt{2p^2 - 4p\sqrt{p} - 2p + 9}} = \frac{2p - 3\sqrt{p} - 1}{\sqrt{2p^2 - 4p\sqrt{p} - 2p + 9}}$$

Er moet gelden $\frac{dL}{dp} = 0$ oftewel $2p - 3\sqrt{p} - 1 = 0$.

Voer in $y_1 = 2x - 3\sqrt{x} - 1$.

De optie nulpunt geeft $x \approx 3,17$.



Dus voor $p \approx 3,17$ is L minimaal.

Bladzijde 131

5 a $\sin(\alpha) = \frac{AS}{5}$ geeft $AS = 5 \sin(\alpha)$

$\cos(\alpha) = \frac{BS}{5}$ geeft $BS = 5 \cos(\alpha)$

De stelling van Pythagoras in $\triangle ADS$ geeft $DS = \sqrt{6^2 - (5 \sin(\alpha))^2} = \sqrt{36 - 25 \sin^2(\alpha)}$.

$BD = BS + DS = 5 \cos(\alpha) + \sqrt{36 - 25 \sin^2(\alpha)}$

b $O(ABCD) = 2 \cdot O(\triangle ABD) = 2 \cdot \frac{1}{2} \cdot (5 \cos(\alpha) + \sqrt{36 - 25 \sin^2(\alpha)}) \cdot 5 \sin(\alpha)$

Voer in $y_1 = (5 \cos(x) + \sqrt{36 - 25 \sin^2(x)}) \cdot 5 \sin(x)$.

De optie maximum geeft $x = 50,19\dots$

Dus de oppervlakte van $ABCD$ is maximaal voor $\alpha \approx 50^\circ$.

Alternatieve uitwerking

De oppervlakte van vierhoek $ABCD$ is maximaal als de oppervlakte van $\triangle ABD$ maximaal is.

Dat is het geval als de hoogte die bij de zijde AB hoort maximaal is. En dat is het geval als $\angle A = 90^\circ$.

Dit geeft $\tan(\alpha) = \frac{AD}{AB} = \frac{6}{5}$ en hieruit volgt $\alpha \approx 50^\circ$.

6 $f(x) = x^4 - x^3 - 9x^2 - 5x$

$f'(x) = 4x^3 - 3x^2 - 18x - 5$

$f''(x) = 12x^2 - 6x - 18$

$f'(1) = -22 < 0$, dus de grafiek van f is dalend voor $x = 1$.

$f''(1) = -12 < 0$, dus de grafiek van f is toenemend dalend voor $x = 1$.

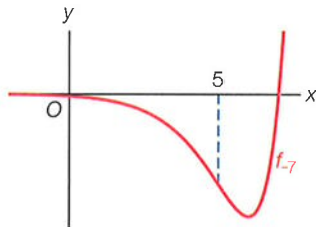
7 $f_a(x) = (x+a)e^x$ geeft $f'_a(x) = 1 \cdot e^x + (x+a) \cdot e^x = (x+a+1)e^x$

$f'_a(x) = (x+a+1)e^x$ geeft $f''_a(x) = 1 \cdot e^x + (x+a+1) \cdot e^x = (x+a+2)e^x$

$f''_a(5) = 0$ geeft $(a+7)e^5 = 0$

$a+7=0$

$a=-7$



Dus voor $a = -7$ gaat de grafiek van f_a bij $x = 5$ over van toenemend dalend naar afnemend dalend.

8 a y is evenredig is met $x \cdot \sqrt[3]{x}$, dus $y = ax \cdot \sqrt[3]{x}$.

$$\left. \begin{array}{l} y = ax \cdot \sqrt[3]{x} \\ x = 8 \text{ en } y = 40 \end{array} \right\} \begin{array}{l} 8a \cdot \sqrt[3]{8} = 40 \\ 8a \cdot 2 = 40 \\ 16a = 40 \\ a = 2\frac{1}{2} \end{array}$$

Dus $y = 2\frac{1}{2}x \cdot \sqrt[3]{x} = 2\frac{1}{2}x^{1\frac{1}{3}}$ en dit geeft $y' = 3\frac{1}{3}x^{\frac{1}{3}}$ en $y'' = 1\frac{1}{9}x^{-\frac{2}{3}}$.

Omdat voor $x > 0$ geldt $x^{\frac{1}{3}} > 0$ en $x^{-\frac{2}{3}} > 0$, is $y' > 0$ en $y'' > 0$.

Dus y is toenemend stijgend voor $x > 0$.

b y is omgekeerd evenredig met $e^{0,1x}$, dus $y = \frac{a}{e^{0,1x}}$.

$$\left. \begin{array}{l} y = \frac{a}{e^{0,1x}} \\ x = 10 \text{ en } y = 4 \end{array} \right\} \begin{array}{l} \frac{a}{e^1} = 4 \\ a = 4e \end{array}$$

Dus $y = \frac{4e}{e^{0,1x}} = 4e^{1-0,1x}$ en dit geeft $y' = 4e^{1-0,1x} \cdot -0,1 = -0,4e^{1-0,1x}$ en

$y'' = -0,4e^{1-0,1x} \cdot -0,1 = 0,04e^{1-0,1x}$.

Omdat geldt $e^{1-0,1x} > 0$, is $y' < 0$ en $y'' > 0$.

Dus y is afnemend dalend.

9 $f(x) = g(x)$ geeft $\frac{-2x+4}{2x-1} = -\frac{2}{3}x + 2\frac{2}{3}$

$$-2x+4 = (2x-1)\left(-\frac{2}{3}x + 2\frac{2}{3}\right)$$

$$-2x+4 = -1\frac{1}{3}x^2 + 5\frac{1}{3}x - 2\frac{2}{3}$$

$$1\frac{1}{3}x^2 - 8x + 6\frac{2}{3} = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-1)(x-5) = 0$$

$$x = 1 \vee x = 5$$

$$f(x) = \frac{-2x+4}{2x-1} = \frac{-(2x-1)-1+4}{2x-1} = -1 + \frac{3}{2x-1}$$

$$O(V) = \int_1^5 \left(-\frac{2}{3}x + 2\frac{2}{3} - \left(-1 + \frac{3}{2x-1} \right) \right) dx = \int_1^5 \left(-\frac{2}{3}x + 3\frac{2}{3} - \frac{3}{2x-1} \right) dx = \left[-\frac{1}{3}x^2 + 3\frac{2}{3}x - 1\frac{1}{2}\ln|2x-1| \right]_1^5$$

$$= -\frac{1}{3} \cdot 25 + 3\frac{2}{3} \cdot 5 - 1\frac{1}{2}\ln(9) - \left(-\frac{1}{3} + 3\frac{2}{3} - 1\frac{1}{2}\ln(1) \right) = -8\frac{1}{3} + 18\frac{1}{3} - 1\frac{1}{2}\ln(3^2) - 3\frac{1}{3} = 6\frac{2}{3} - 3\ln(3)$$

10 a $y = 4 \ln(x)$ geeft $\ln(x) = \frac{1}{4}y$

$$x = e^{\frac{1}{4}y}$$

$$O(V) = \int_0^p e^{\frac{1}{4}y} dy = \left[4e^{\frac{1}{4}y} \right]_0^p = 4e^{\frac{1}{4}p} - 4$$

$$O(V) = O(W) \text{ oftewel } O(V) = \frac{1}{2}p^2 \text{ geeft } 4e^{\frac{1}{4}p} - 4 = \frac{1}{2}p^2.$$

Voer in $y_1 = 4e^{\frac{1}{4}x} - 4$ en $y_2 = \frac{1}{2}x^2$.

De optie snijpunt geeft $x \approx 2,963...$, dus $p \approx 2,96$.

b $y = 4 \ln(x)$ geeft $\ln(x) = \frac{1}{4}y$

$$x = e^{\frac{1}{4}y}$$

$$x^2 = e^{\frac{1}{2}y}$$

$$I(L) = \pi \int_0^p x^2 dy = \pi \int_0^p e^{\frac{1}{2}y} dy = \pi \left[2e^{\frac{1}{2}y} \right]_0^p = \pi(2e^{\frac{1}{2}p} - 2)$$

$$I(L) = I(M) \text{ oftewel } I(L) = \frac{1}{2} \cdot I(\text{cilinder}) \text{ geeft } \pi(2e^{\frac{1}{2}p} - 2) = \frac{1}{2}\pi p^2 \cdot p$$

$$2e^{\frac{1}{2}p} - 2 = \frac{1}{2}p^3$$

Voer in $y_1 = 2e^{\frac{1}{2}x} - 2$ en $y_2 = \frac{1}{2}x^3$.

De optie snijpunt met $1\frac{1}{2} < x < 8$ geeft $x \approx 1,801...$

Dus $p \approx 1,80$.